
MR2053457 (2005g:11071)
Dummigan, Neil (4-SHEF-PM); **Stein, William** (1-HRV); **Watkins, Mark** (1-PAS)
Constructing elements in Shafarevich-Tate groups of modular motives. (English summary)
Number theory and algebraic geometry, 91–118, *London Math. Soc. Lecture Note Ser.*, 303, *Cambridge Univ. Press, Cambridge*, 2003.
11F33 (11F67 11F80 11G18)

The authors generalize some of the ideas of J. E. Cremona and B. Mazur [see A. Agashé and W. A. Stein, Math. Comp. **74** (2005), no. 249, 455–484 (electronic);; MR2085902 (2005g:11119) (Appendix)] for producing nontrivial elements of the Tate-Shafarevich groups to the case of modular forms of higher weight.

More precisely, let f and g be newforms of even weight $k > 2$ on $\Gamma_0(N)$ satisfying $L(f, k/2) \neq 0 = L(g, k/2)$. If f is congruent to g modulo a prime ideal $\mathfrak{q}$ not dividing $2Nk!$, then the authors show, under suitable additional hypotheses, that: (A) $\mathfrak{q}$ divides the algebraic part of $L(f, k/2)$ [cf. V. Vatsal, Duke Math. J. **98** (1999), no. 2, 397–419; MR1695203 (2000g:11032)]; (B) $\mathfrak{q}$ does not divide the Tamagawa factors ("the fudge factors") in the Bloch-Kato conjecture for $L(f, k/2)$; (C) denoting by $V_{\mathfrak{q}}$ (resp. $V'_{\mathfrak{q}}$) the $\mathfrak{q}$-adic Galois representation associated to f (resp. to g), then the rank of the $\mathfrak{q}$-torsion of the generalized Tate-Shafarevich group associated to f at $k/2$ is at least equal to $\dim H^1_f(\mathbf{Q}, V'_{\mathfrak{q}}(k/2))$ (the latter being conjecturally equal to the order of vanishing of $L(g, s)$ at $s = k/2$). These results provide evidence for the $\mathfrak{q}$-part of the Bloch-Kato conjecture for $L(f, k/2)$.

The authors also include a table of numerical experiments, including several examples in which not all of the additional assumptions are satisfied.

{Reviewer's remarks: (1) In the last sentence before Theorem 6.1: the vanishing of $H^1_f(\mathbf{Q}, V_{\mathfrak{q}}(k/2))$ follows from $L(f, k/2) \neq 0$, by the work of K. Katō [Astérisque No. 295 (2004), ix, 117–290; MR2104361]. (2) The proof of Theorem 6.1 (case 3) is complicated by the fact that the authors seem to be unaware of a notorious misprint in the article of S. J. Bloch and Katō [in *The Grothendieck Festschrift, Vol. I*, 333–400, Progr. Math., 86, Birkhäuser Boston, Boston, MA, 1990; MR1086888 (92g:11063)]: in Lemma 4.5, H^1_e should be replaced by H^1_f. (3) It is not yet known how to associate a Chow motive to a modular form g of weight $k > 2$, which means that the object $\mathrm{CH}^{k/2}(M_g)$ is not defined.}

{For the entire collection see MR2053451 (2004k:00024)}

Reviewed by *Jan Nekovář*

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