Homework Assignment 1
Due September 26, 2001

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Instructions: *Please work in groups,* and acknowledge those you work with in your write up. Some of the problem below, such as "factor a number" can be quickly done with a computer. Feel free to do so, unless otherwise stated.

1. Let p be a prime number and r and integer such that $1 \leq r < p$. Prove that p divides the binomial coefficient
$$\frac{p!}{r!(p-r)!}.$$
You may not assume that this coefficient is a integer.

2. Compute the following gcd's using a pencil and the Euclidean algorithm:
$$\gcd(15, 35), \quad \gcd(247, 299), \quad \gcd(51, 897), \quad \gcd(136, 304)$$

3. Using mathematical induction to prove that
$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2},$$
then find a formula for
$$1 - 2 + 3 - 4 + \cdots \pm n = \sum_{a=1}^{n} (-1)^{a-1} a.$$

4. What was the most recent prime year? I.e., which of $2001, 2000, \ldots$ was it?

5. Use the Euclidean algorithm to find integers x, $y \in \mathbb{Z}$ such that
$$2261x + 1275y = 17.$$
[I did not tell you how to do this; see §1.8 of Davenport's book.]

6. Factor the year that you should graduate from Harvard as a product of primes. E.g., frosh answer $2005 = 5 \times 401$.

7. Write a PARI program to print "Hello Kitty" five times.

8. Let $f(x) \in \mathbb{Z}[x]$ be a polynomial with integer coefficients. Formulate a conjecture about when the set $\{f(a) : a \in \mathbb{Z} \text{ and } f(a) \text{ is prime }\}$ is infinite. Give computational evidence for your conjecture.

9. Is it easy or hard for PARI to compute the gcd of two random 2000-digit numbers?

10. Prove that there are infinitely many primes of the form $6x - 1$.

11. (a) Use PARI to compute

$$\pi(2001) = \#\{ \text{ primes } p \leq 2001\}.$$

(b) The prime number theorem predicts that $\pi(x)$ is asymptotic to $x/\log(x)$. How close is $\pi(2001)$ to $2001/\log(2001)$?