Math 581e, Fall 2012, Homework 4

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Due: Friday, October 26, 2012

There are 4 problems. Turn your solutions in Friday, October 26, 2012 in class. You may work with other people and can find the LaTeX of this file at http://wstein.org/edu/2012/ant/hw/. If you use Sage to solve a problem, include your code in your solution. I have office hours 12:30–2:00 on Wednesdays in Padelford C423.

1. Let I be a nonzero integral ideal in the ring of integers $\mathcal{O}_K$ of a number field K. Prove that $I^{-1} = \{\alpha \in K : \alpha I \subset I\}$. [In class, we proved exactly this statement in the case when I is a prime ideal.]

2. Give the most elementary proof you can that the abelian group of fractional ideals in the ring of integers of a number field is torsion free. [Obviously, this follows from the theorem that every fractional ideal can be written uniquely as a product of prime ideals. However, that theorem took a lot of work to prove. Find an argument that uses less total "proof energy".]

3. Explicitly factor the fractional ideal $(2/3)\mathcal{O}_K$ for K each of the following fields: $K = \mathbb{Q}, \mathbb{Q}(\sqrt{5}), \mathbb{Q}(\sqrt{2}), \mathbb{Q}(\sqrt[3]{2}), \mathbb{Q}(\sqrt{2}, \sqrt{3})$. [You can use Sage if you want.]

4. Come up with an idea for what you might do your final project on. (You can change based on my feedback, etc.)

   (a) Title of your final project:

   (b) Abstract of your final project (one paragraph):

   (c) Other people you might collaborate with on your final project (not required).

   (d) Estimated amount of time you intend to spend on your final project.