

Math 1062, Spring 2012, The Midterm

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Due: Friday, Feb 17, 2012

- **Do not** work with anybody else on this midterm. **You may** search anywhere online for help, documentation, etc.
- You may use anything in Sage to do the computations below. I will grade this purely based on your function working on my test data and nothing else.
- Your solution will be a Sage worksheet that contains a cell that defines a function `f`. Email this to wstein@uw.edu by Friday Feb 17 at midnight with “1062 midterm” in the subject.

1 The Problem

Write a function f that takes as input a list v of Sage integers and outputs a dictionary d with 10 keys (or 13 keys if you are a graduate student) and values as given below. I will run your function on several inputs v (that will *all* be lists of Sage integers).

1. **prod**: product of entries in v
2. **sum**: sum of entries in v
3. **count**: the number of *distinct* (i.e., different) entries in v
4. **mean**: the mean (average) of the numbers in v
5. **std**: the standard deviation, using the definition where you divide by $\text{len}(v)-1$; set this to 0 if $\text{len}(v)\leq 1$.
6. **all_prime**: True if every element in v is a prime; otherwise, False.
7. **all_composite**: True if every element in v is not prime; otherwise, False. For the purposes of this exam, just define *composite* to be “not prime”, though typically one defines a composite number to be a non-prime integer ≥ 2 .
8. **diffs**: the list $v[i]-v[i-1]$ of differences of successive elements of v
9. **quos**: the list $v[i]/v[i-1]$ of quotients of successive elements of v ; or, if any $v[i-1] == 0$, set $d['quos']$ to None.
10. **num_even**: the number of even elements of v .
11. (Graduate Students only) **gcd**: greatest common divisor of entries in v .
12. (Graduate Students only) **num_3mod4**: number of elements of v that are congruent to 3 modulo 4.

13. (Graduate Students only) **crt**: Any integer n such that $n \equiv i \pmod{v[i]}$ for all i with $0 \leq i < \text{len}(v)$, if such an n exists. If there is no such n , set `d['crt']`=None.

2 Examples and Hints

Your function will look like this:

```
def f(v):
    d = {}

    # raises TypeError if anything in v not an integer.
    v = [ZZ(a) for a in v]

    # product of entries in v
    d['prod'] = prod(v)

    # ... fill this in ...

    return d
```

Here are some example inputs and outputs. Don't worry about the order in which the key:value pairs are displayed:

```
sage: f([])
{'count': 0, 'std': 0, 'gcd': 0, 'crt': 0, 'diffs': [], 'sum': 0,
 'num_even': 0, 'all_prime': True, 'quos': [], 'num_3mod4': 0, 'prod': 1,
 'all_composite': True, 'mean': NaN}

sage: f([1])
{'count': 1, 'std': 0, 'gcd': 1, 'crt': 0, 'diffs': [], 'sum': 1,
 'num_even': 0, 'all_prime': False, 'quos': [], 'num_3mod4': 0, 'prod': 1,
 'all_composite': True, 'mean': 1}

sage: f(prime_range(10))
{'count': 4, 'std': 1/2*sqrt(59/3), 'gcd': 1, 'crt': 52,
 'diffs': [1, 2, 2], 'sum': 17, 'num_even': 1, 'all_prime': True,
 'quos': [3/2, 5/3, 7/5], 'num_3mod4': 2, 'prod': 210,
 'all_composite': False, 'mean': 17/4}

sage: f([-4..4])
{'count': 9, 'std': sqrt(15/2), 'gcd': 1, 'crt': None,
 'diffs': [1, 1, 1, 1, 1, 1, 1, 1], 'sum': 0, 'num_even': 5,
 'all_prime': False, 'quos': None, 'num_3mod4': 2, 'prod': 0,
 'all_composite': False, 'mean': 0}

sage: f([0,2,...,10])
{'count': 6, 'std': sqrt(14), 'gcd': 2, 'crt': None,
 'diffs': [2, 2, 2, 2, 2], 'sum': 30, 'num_even': 6,
 'all_prime': False, 'quos': None, 'num_3mod4': 0, 'prod': 0,
 'all_composite': False, 'mean': 5}

sage: f([0,1,0,1,0,1])
{'count': 2, 'std': sqrt(3/10), 'gcd': 1, 'crt': None,
 'diffs': [1, -1, 1, -1, 1], 'sum': 3, 'num_even': 3, 'all_prime': False,
 'quos': None, 'num_3mod4': 0, 'prod': 0, 'all_composite': True, 'mean': 1/2}

sage: f([1]*10)
{'count': 1, 'std': 0, 'gcd': 1, 'crt': 0,
 'diffs': [0, 0, 0, 0, 0, 0, 0, 0, 0], 'sum': 10, 'num_even': 0,
 'all_prime': False, 'quos': [1, 1, 1, 1, 1, 1, 1, 1, 1],
 'num_3mod4': 0, 'prod': 1, 'all_composite': True, 'mean': 1}

sage: set_random_seed(0); f([ZZ(randint(0,10)) for i in range(10)])
{'count': 8, 'std': 2/3*sqrt(107/5), 'gcd': 1, 'crt': None,
 'diffs': [4, -5, 3, 2, -3, 5, -7, 9, -3], 'sum': 38, 'num_even': 4,
 'all_prime': False, 'quos': None, 'num_3mod4': 2, 'prod': 0,
 'all_composite': False, 'mean': 19/5}
```