# Math 1062, Spring 2012, Homework 5

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**Due: Friday, Feb 24, 2012**

- You are encouraged to work with other people on homework; thank them explicitly in your write up.
- You can find the LaTeX of this file at http://wstein.org/edu/2012/1062/hw/.
- I will **NOT** have office hours 11am–2pm in Padelford C423 on Thursday Feb 23, since I will be in San Diego. You can still email me at wstein@gmail.com or the list at uw-sage-2012@googlegroups.com for help. I will often cc a sanitized version of my answer to the list, so everybody benefits.
- Your solution will be a Sage worksheet.

## 1 Homework

1. Let $A_n$ be the $n \times n$ matrix whose zero-based $(i, j)$ entry is the rational number $(i^2 - 21i + 110)/(i + j + 1)$, so for $n = 4$, we have

$$A_4 = \begin{pmatrix} 110 & 55 & \frac{110}{3} & \frac{55}{2} \\ 45 & 30 & \frac{45}{2} & 18 \\ 24 & 18 & \frac{72}{5} & 12 \\ 14 & \frac{56}{5} & \frac{28}{3} & 8 \end{pmatrix}$$

   (a) For $n = 1, 2, \ldots, 20$, what is the nullity of $A_n$ (i.e., the dimension of the kernel of $A_n$)?

   (b) Compute an explicit basis for the left kernel of $A_{11}$, i.e., for the set of vectors $v$ such that $vA_{11} = 0$.

   (c) Compute an explicit basis for the row space of $A_{11}$, i.e., the span of the rows.

   (d) (Graduate Students) Do you think nullity of $A_n$ is ever $\geq 3$? Give evidence either way, or an argument.

2. Based on testing, do you think Sage can compute the determinant of most any random $500 \times 500$ matrix with single digit integer entries in less than a minute?

3. (Graduate students only) Explain as much as you can about how Sage computes the determinant of a matrix with integer entries. (You'll probably have to dig into source code.)