Exercise Set 5:
Galois Cohomology

Math 582e, Winter 2010, University of Washington

Due Wednesday, February 24, 2010

1. (a) Suppose $f : A \to A$ is a homomorphism and A is a finite abelian group. Prove that $\# \ker(f) = \# \operatorname{coker}(f)$.

   (b) Is it necessarily the case that $\ker(f) \approx \operatorname{coker}(f)$ as abelian groups?

2. Give examples of central simple $\mathbb{Q}$-algebras that correspond to elements of order 1, 2, and 3 in $\mathrm{Br}_{\mathbb{Q}}$. (Hint: Orders 1 and 2 are very easy. I'm not so sure how to give an explicit element of order 3, so do some digging.)

3. Give examples of genus one curves X over $\mathbb{Q}$ that correspond to elements of order 1, 2, and 3 in the $\mathrm{H}^1(\mathbb{Q}, E)$, where $E = \mathrm{Jac}(X)$.