Math 581d, Fall 2010, Homework 8

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Due: November 24, 2010

Do the following 4 problems, and turn them in by email (`wstein@gmail.com`) on Wednesday, November 24, 2010. As usual, you can find the latex of this file at `http://wstein.org/edu/2010/581d/hw/`.

This will be the last homework assignment of this course. All the future work for credit that you'll do in this class after this assignment will focus entirely on your final project.

1. Create new Sage parent and element classs that model $\mathbf{Q}(\sqrt{5}) = \{a+b\sqrt{5} : a, b \in \mathbf{Q}\}$ and its elements, following the examples and documentation at `http://sagemath.org/doc/reference/coercion.html`. Obviously, try to document your implementation as much as you can, including doctests, INPUT and OUTPUT blocks, etc.

2. Benchmark your implementation of the ring R from problem 1:

   (a) How fast is addition, multiplication, division, and subtraction for the two elements $\alpha = 17 + \sqrt{5}/7$ and $\beta = 2/9 - \sqrt{5}/18$?

   (b) How fast is addition, multiplication, division, and subtraction for the two elements $\alpha = 17 + \sqrt{5}/7 \in R$ and $\beta = 2/9 \in \mathbf{Q}$. Here you should define β to be $2/9$ with `beta.parent()` outputing the rational field, not your new ring R. The point is that the benchmarks should be slowed down by the automatic coercion that happens when you do arithmetic with α and β.

3. Rewrite your elements from Problem 1 using one of the Cython implementations of elements you have from last week's homework. You are allowed to use one that represents elements using long's (i.e., it can overflow), just to see how fast you can make it. Then benchmark exactly as above (both with elements of the ring and with one element not in the ring).

4. Do problem 1 above for another algebraic structure (a ring, vector space, whatever) of your choosing other than $\mathbf{Q}(\sqrt{5})$.