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# 480 - 20090515 - combinatorics 3 lab

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## Math 480: Sloane's Tables of Integer Sequences (LAB)

### 2009-05-15

### Integer Sequences

An integer sequence  $a_0, a_1, a_2, \dots$ , is simply a sequence of integers. It's as simple as 1, 2, 3, 4, 5, 6, ...!

#### Examples:

1. Natural numbers
2. The prime numbers
3. The Fibonacci numbers
4. Bell numbers
5. Triangle numbers ( $1 + 2 + \dots + n$ )
6. Digits of  $\pi$

**Neil Sloane** has devoted tremendous energy to cataloguing, indexing, organizing, and making easily available and searchable an *encyclopedia* of integer sequences, which he started in the 1960s as a grad student, and which has been online since the mid 1990s.



**BIO:** *Neil J. A. Sloane grew up in Australia and received his masters and doctorate degrees from Cornell University. He joined AT&T Bell Laboratories in 1969 and is now at AT&T Labs-Research in New Jersey. Dr. Sloane has co-authored more than 10 books, including books on such diverse subjects as integer sequences and rock climbing. His On-Line Encyclopedia of Integer Sequences receives thousands of visits each day. Additionally, the prolific Dr. Sloane has authored 279 publications, which include papers on coding theory, spherical codes and designs, lattices and quadratic forms, packings in Grassmanian spaces, combinatorics, cryptography, and integer sequences.*

## Neil Sloane's Online Encyclopedia of Integer Sequences

Contains **158602** sequences and cited by over 1200 papers.

<http://www.research.att.com/~njas/sequences>

Listen to a podcast about it online at some point: <http://mathfactor.uark.edu/2008/05/05/dw-the-online-encyclopedia-of-integer-sequences/>

The OEIS is really amazing. You can even *listen* to integer sequences, played on one of dozens of instruments. Very neat.

**Problem:** Go to Sloane's tables and lookup some integer sequences.

1. Browse around for a few minutes.
2. Try typing in some sequence that comes from mathematics and means something to you.

3. Lookup all sequences listed at the top of this worksheet in the initial examples list.




## Sage Interface to Online Tables

Sage includes some commands that automatically work with Sloane's online tables. These require internet access:

1. `sloane_find(list)` -- given a list of integers, finds all sequences in the online table that match it.
2. `sloane_sequence(n)` -- return the nth sequence in Sloane's tables

These functions, plus some others that support an optional offline database, were all written by Steven Sivek as an undergraduate student project.

```
sloane_find([1..77])
```

```
Searching Sloane's online database...
[[27, 'The natural numbers. Also called the whole numbers, the
counting numbers or the positive integers.', [1, 2, 3, 4, 5, 6, 7,
8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,
25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58,
59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75,
76, 77]], [1477, 'The nonnegative integers.', [0, 1, 2, 3, 4, 5, 6,
7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,
25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58,
59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75,
76, 77]], [33619, 'Undulating numbers (of form abababab... in base
10).', [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33,
34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50,
51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67,
68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84,
85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101,
111, 121, 131, 141, 151]], [130734, 'List of numbers of cents you
can have in US coins without having change for a dollar.', [1, 2, 3,
4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
```

```
22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38,
39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,
56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72,
73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89,
90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109,
115, 116, 117, 118, 119]]]
```

```
sloane_sequence(27)
```

```
Searching Sloane's online database...
```

```
[27, 'The natural numbers. Also called the whole numbers, the
counting numbers or the positive integers.', [1, 2, 3, 4, 5, 6, 7,
8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24,
25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58,
59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75,
76, 77]]]
```

```
v = [n*(n+1)/2 for n in [1..20]]
print v
sloane_find(v)
```

```
[1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136,
153, 171, 190, 210]
Searching Sloane's online database...
[[217, 'Triangular numbers: a(n) = C(n+1,2) = n(n+1)/2 =
0+1+2+...+n.', [0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91,
105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351,
378, 406, 435, 465, 496, 528, 561, 595, 630, 666, 703, 741, 780,
820, 861, 903, 946, 990, 1035, 1081, 1128, 1176, 1225, 1275, 1326,
1378, 1431]], [109811, 'Triangular numbers (A000217) at Levenshtein
distance 1 from another triangular number when considered as a
decimal string.', [0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78,
91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 300, 378, 406, 435,
465]], [105340, 'a(n) = n*(n+1)/2 mod 2048.', [0, 1, 3, 6, 10, 15,
21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210,
231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561,
595, 630, 666, 703, 741, 780, 820, 861, 903, 946, 990, 1035, 1081,
1128, 1176, 1225, 1275, 1326, 1378, 1431, 1485]], [25747, 'Index of
10^n within sequence of numbers of form 9^i*10^j.', [1, 3, 6, 10,
15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190,
210, 231, 254, 278, 303, 329, 356, 384, 413, 443, 474, 506, 539,
573, 608, 644, 681, 719, 758, 798, 839, 881, 924, 969, 1015, 1062,
1110, 1159, 1209, 1260, 1312, 1365, 1419, 1474, 1530, 1587]],
[105337, 'a(n) = n*(n+1)/2 mod 256.', [0, 1, 3, 6, 10, 15, 21, 28,
36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253,
20, 44, 69, 95, 122, 150, 179, 209, 240, 16, 49, 83, 118, 154, 191,
229, 12, 52, 93, 135, 178, 222, 11, 57, 104, 152, 201, 251, 46, 98,
151, 205, 4, 60, 117, 175, 234, 38, 99]], [105338, 'a(n) = n*(n+1)/2
mod 512.', [0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105,
120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378,
406, 435, 465, 496, 16, 49, 83, 118, 154, 191, 229, 268, 308, 349,
391, 434, 478, 11, 57, 104, 152, 201, 251, 302, 354, 407, 461, 4,
60, 117, 175, 234]], [105339, 'a(n) = n*(n+1)/2 mod 1024.', [0, 1,
3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153,
```

```
171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465,
496, 528, 561, 595, 630, 666, 703, 741, 780, 820, 861, 903, 946,
990, 11, 57, 104, 152, 201, 251, 302, 354, 407, 461, 516, 572,
629]], [89594, 'Alternating sum of squares to n.', [-1, 3, -6, 10,
-15, 21, -28, 36, -45, 55, -66, 78, -91, 105, -120, 136, -153, 171,
-190, 210, -231, 253, -276, 300, -325, 351, -378, 406, -435, 465,
-496, 528, -561, 595, -630, 666, -703, 741, -780, 820, -861, 903,
-946, 990, -1035, 1081, -1128, 1176, -1225, 1275]]]
```



**Problem:** Use the Sage `sloane_sequence` command to find each of the sequences in Sloane's tables that are listed at the very top of this worksheet.





## Sage implementation of code to compute about 150 of the most popular sequences

Jaap Spies (of Holland) implemented functions in Sage to compute about 150 of the most popular Sloane sequences.

```
sloane.A000005
```

```
The integer sequence tau(n), which is the number of divisors of n.
```

```
sloane.A000005(10)
```

```
4
```

```
sloane.A000005.list(20)
```

```
[1, 2, 2, 3, 2, 4, 2, 4, 3, 4, 2, 6, 2, 4, 4, 5, 2, 6, 2, 6]
```

**Problem:** Explore the sloane implementations in Sage.

1. Type **sloane.<tab>** to see a list of all the sequences that are implemented.
2. Choose a sequence and see what its description is.
3. Try to compute the first 100 terms of your sequence (use the **.list(100)** command).
4. Do something like **sloane.A000005.\_eval??** to see the Sage source code that is responsible for computing your sequence.
5. Browse around the source code for all the sloane functions here: [/src/combinat/sloane\\_functions.py](#)
6. Which of the integer sequences enumerated at the top of this worksheet is implemented in Sage (in the sloane.<tab> function)?

  

  

  

  


## Problem Solving

One powerful strategic way to use Sloane's tables is to quickly solve certain types of problems when you have no clue what to do.

1. Compute a list of integers that come up naturally in your problem.
2. Search for them with the `sloane_find` command in Sage.
3. If anything potentially interesting pops up, go to the website and get extensive additional information, and hopefully a solution to your problem.

Example: Suppose you want to find a formula for  $1^2 + 2^2 + \dots + n^2$ .

Solution: Try computing the first few terms of this sequence, and look it up in Sloane's tables.

```
v = [sum(a^2 for a in [1..n]) for n in [1..20]]
v
[1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015,
1240, 1496, 1785, 2109, 2470, 2870]
```

```
sloane_find(v)
Searching Sloane's online database...
[[330, 'Square pyramidal numbers: 0^2+1^2+2^2+...+n^2 =
n(n+1)(2n+1)/6.', [0, 1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506,
650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870, 3311, 3795,
4324, 4900, 5525, 6201, 6930, 7714, 8555, 9455, 10416, 11440, 12529,
13685, 14910, 16206, 17575, 19019, 20540, 22140, 23821, 25585,
27434, 29370]], [109678, 'Sequence and first differences include all
square numbers exactly once.', [1, 5, 14, 30, 55, 91, 140, 204, 285,
385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870, 3311,
3795, 4324, 4900, 5525, 6201, 6930, 7714, 8555, 9455, 10416, 11440,
12529, 13685, 14910, 16206, 17575, 19019, 20540, 22140, 23821,
25585]]]
```

And we thus get the formula  $n(n+1)(2n+1)/6$ , just like that. If you want more information, go to the website. There you might learn such facts as that this sequence:

The sequence contains exactly one square greater than 1, namely 4900 (according to Gardner). - Jud McCranie (j.mccranie(AT)comcast.net), Mar 19 2001, Mar 22 2007

```
v = [sum(a^2 for a in [1..n]) for n in [1..100]]
v
[1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015,
1240, 1496, 1785, 2109, 2470, 2870, 3311, 3795, 4324, 4900, 5525,
6201, 6930, 7714, 8555, 9455, 10416, 11440, 12529, 13685, 14910,
16206, 17575, 19019, 20540, 22140, 23821, 25585, 27434, 29370,
31395, 33511, 35720, 38024, 40425, 42925, 45526, 48230, 51039,
53955, 56980, 60116, 63365, 66729, 70210, 73810, 77531, 81375,
85344, 89440, 93665, 98021, 102510, 107134, 111895, 116795, 121836,
127020, 132349, 137825, 143450, 149226, 155155, 161239, 167480,
173880, 180441, 187165, 194054, 201110, 208335, 215731, 223300,
231044, 238965, 247065, 255346, 263810, 272459, 281295, 290320,
299536, 308945, 318549, 328350, 338350]
```

```
[a for a in v if a.is_square()]
[1, 4900]
```

**Problem:**

1. Use Sloane's tables to find a closed formula for the sum of the cubes of the first  $n$  integers:  $1^3 + 2^3 + \dots + n^3$ .
2. Use Sloane's tables to find out what is so special about the primes 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97, ...
3. Use Sloane's tables to figure out what mysterious transcendental number has decimal expansion 7.38905609893065.