Math 480 (Spring 2007): Homework 5

Due: Monday, April 30

There are 3 problems. Each problem is worth 6 points and parts of multipart problems are worth equal amounts. You may work with other people and use a computer, unless otherwise stated. Acknowledge those who help you.

1. Encode the message `NUMBER THEORY` as a single number in base 27, where 0 corresponds to a space, A to 1, B to 2 and so on.

2. How many solutions does the following system of congruences have?

$$\begin{aligned} x &\equiv 3 \pmod{18} \\ x &\equiv 2 \pmod{3} \\ x &\equiv 1 \pmod{6} \end{aligned}$$

3. In class I mentioned the famous open problem that there are infinitely many primes p such that $(p-1)/2$ is also prime. Is it reasonable to conjecture that there are infinitely many primes p such that $p \equiv 1 \pmod{3}$ and $(p-1)/3$ is prime?