Math 168: Homework Assignment 4

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Due: Wednesday, Oct 26, 2005

The problems have equal point value, and multi-part problems are of the same value.

1 Problems

1. Use the enumeration of Pythagorean triples (a, b, c) with $a^2+b^2 = c^2$ to create a program that lists all square-free congruent numbers. These will, of course, not be in increasing order! You do not know when a given congruent number will appear on your list.

2. In this problem you will create a procedure to output *all* triangles with rational side lengths and area 219.

   (a) Let E be the elliptic curve $y^2 = x^3 - 219^2x$. Compute generators for the Mordell-Weil group $E(\mathbb{Q})$ using a computer program (e.g., SAGE, Magma, mwrank, etc.) [Hint: E has rank 2.]

   (b) Describe how to systematically list *all* elements of $E(\mathbb{Q})$ with $y \neq 0$.

   (c) Use Prop 6.5.7 from the class notes and your answer to the above question to systematically list all triangles with rational side lengths and area 219.