Graphs of L-Series and Modular Forms Attached to Elliptic Curves

William Stein and Kevin Grosvenor

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1 The Elliptic Curve $11A$
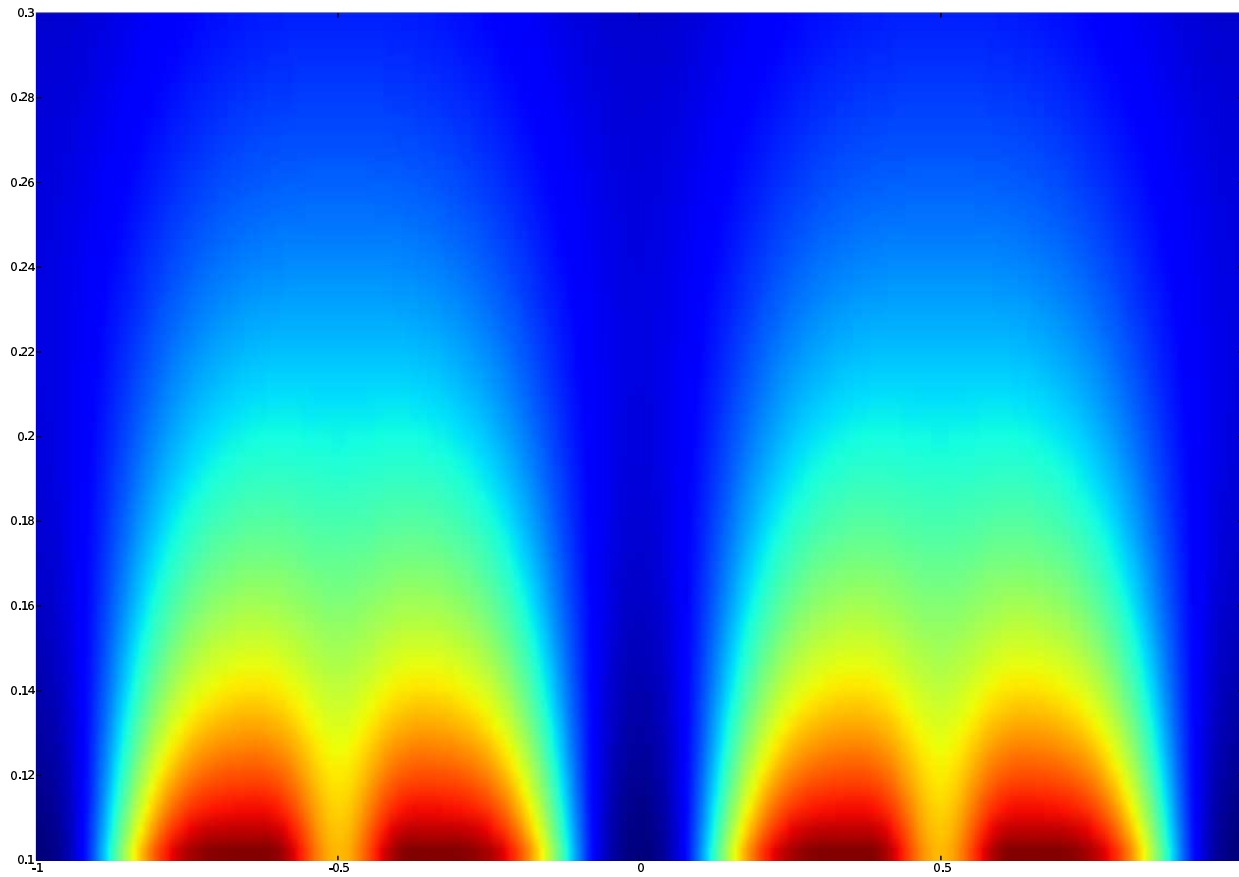
This is an elliptic curve of rank 0.

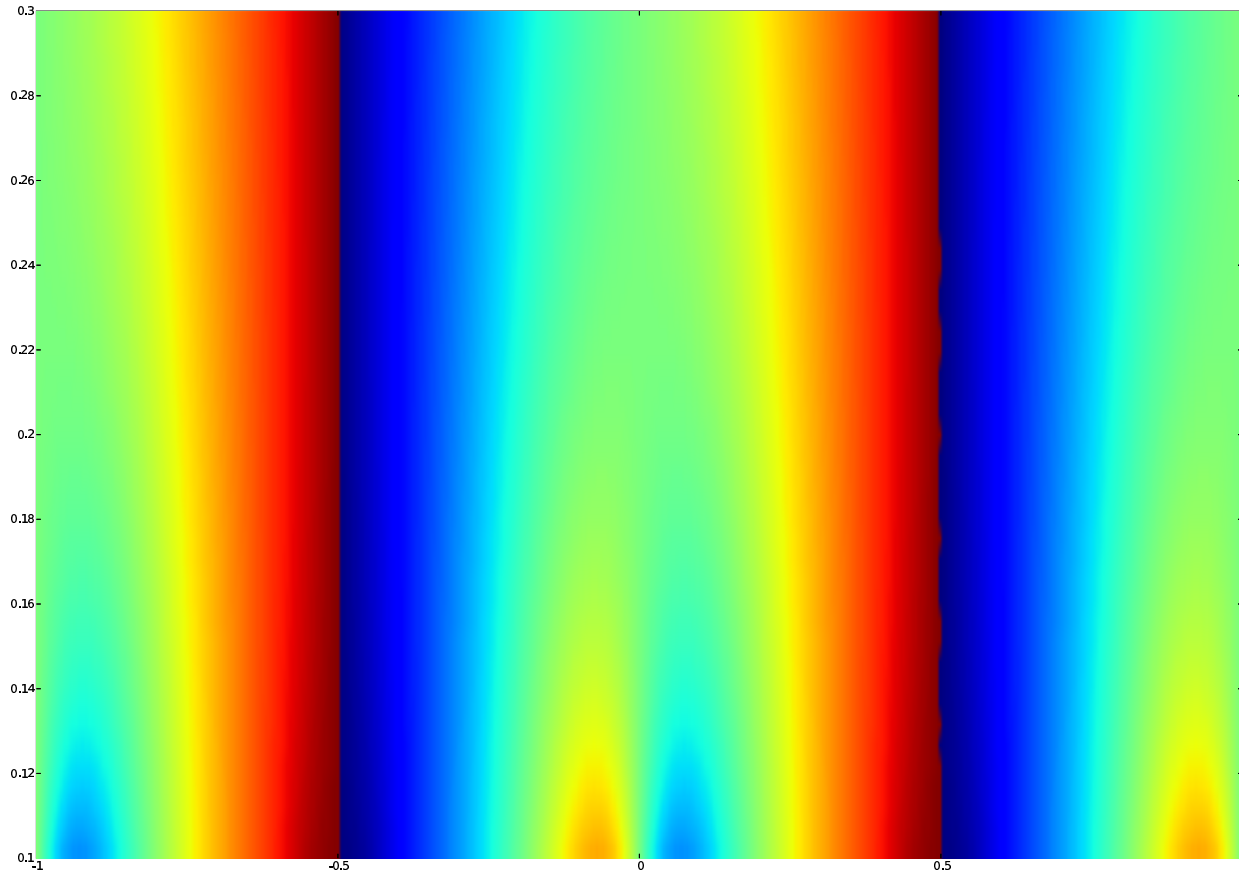$$y^2 + y = x^3 - x^2 - 10x - 20$$

1.1   The Modular Form

The modular form is

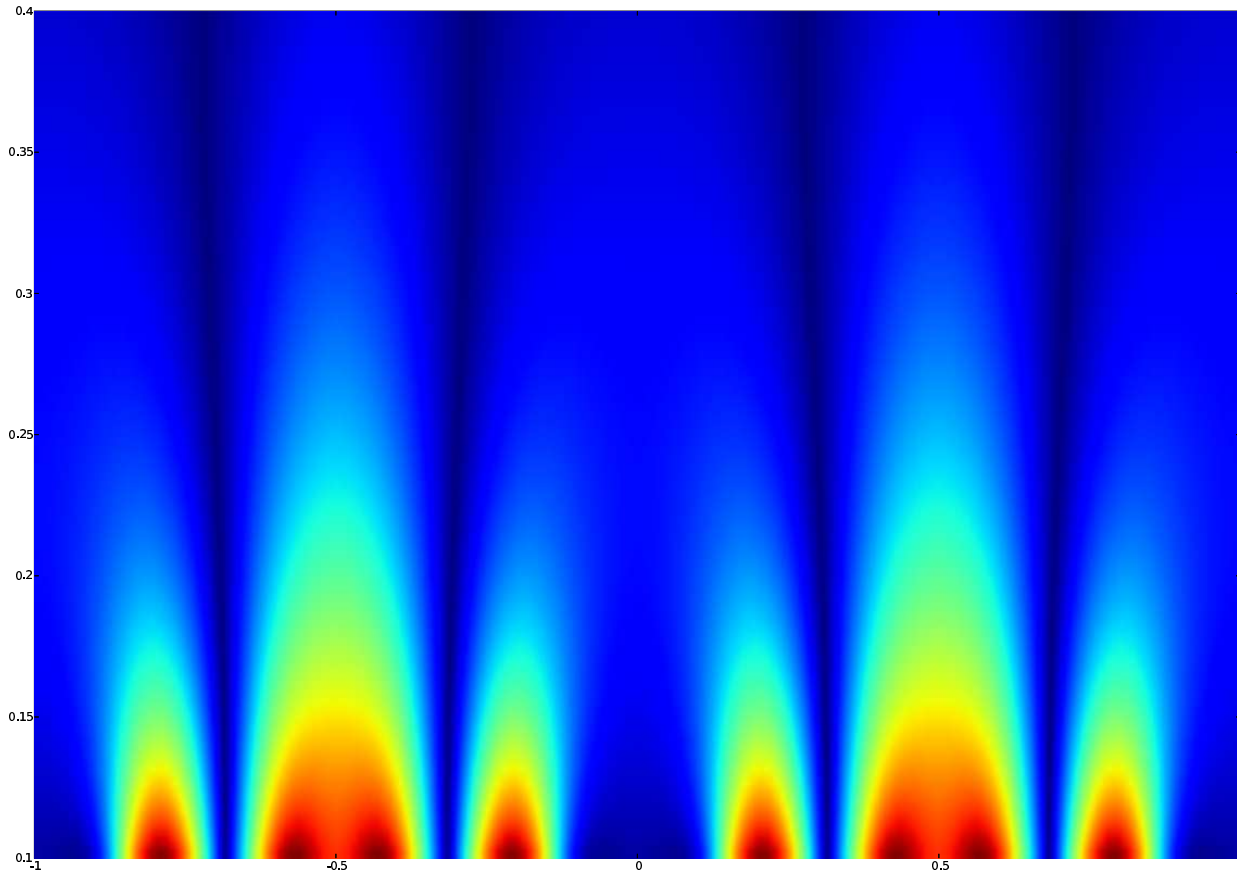$$q - 2q^2 - q^3 + 2q^4 + q^5 + 2q^6 - 2q^7 - 2q^9 + \cdots.$$

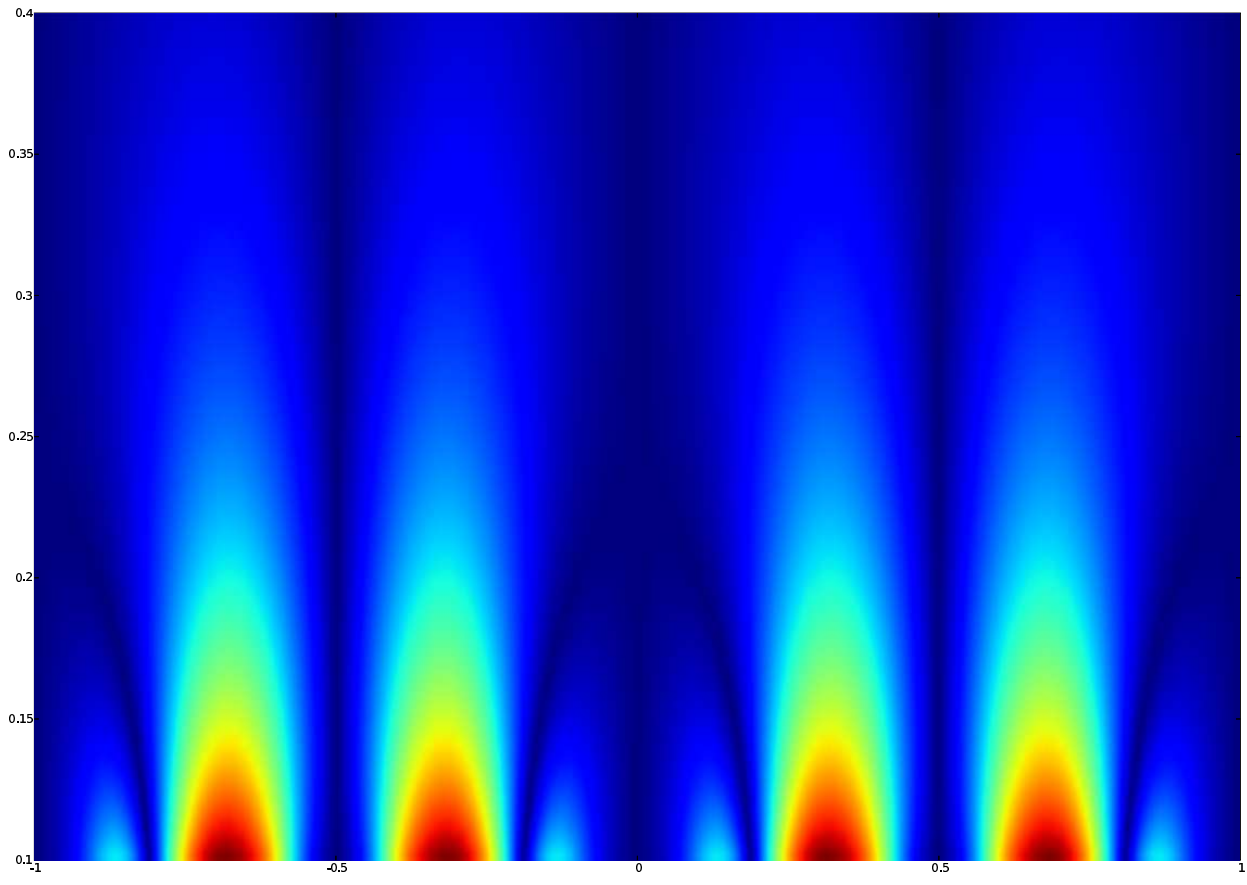Absolute Value of Elliptic Curve 11A Modular Form

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Argument of Elliptic Curve 11A Modular Form

Absolute Real Part of 11A Curve Modular Form

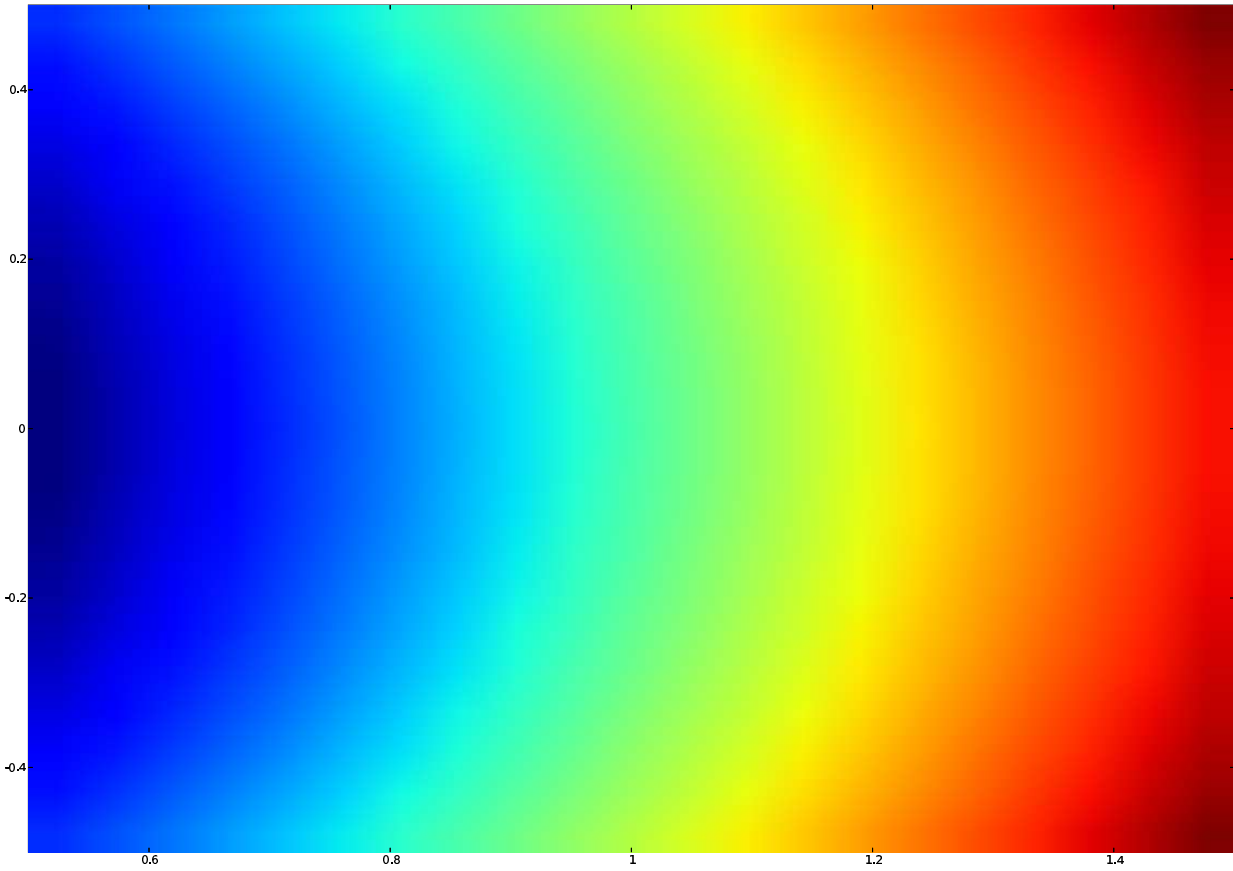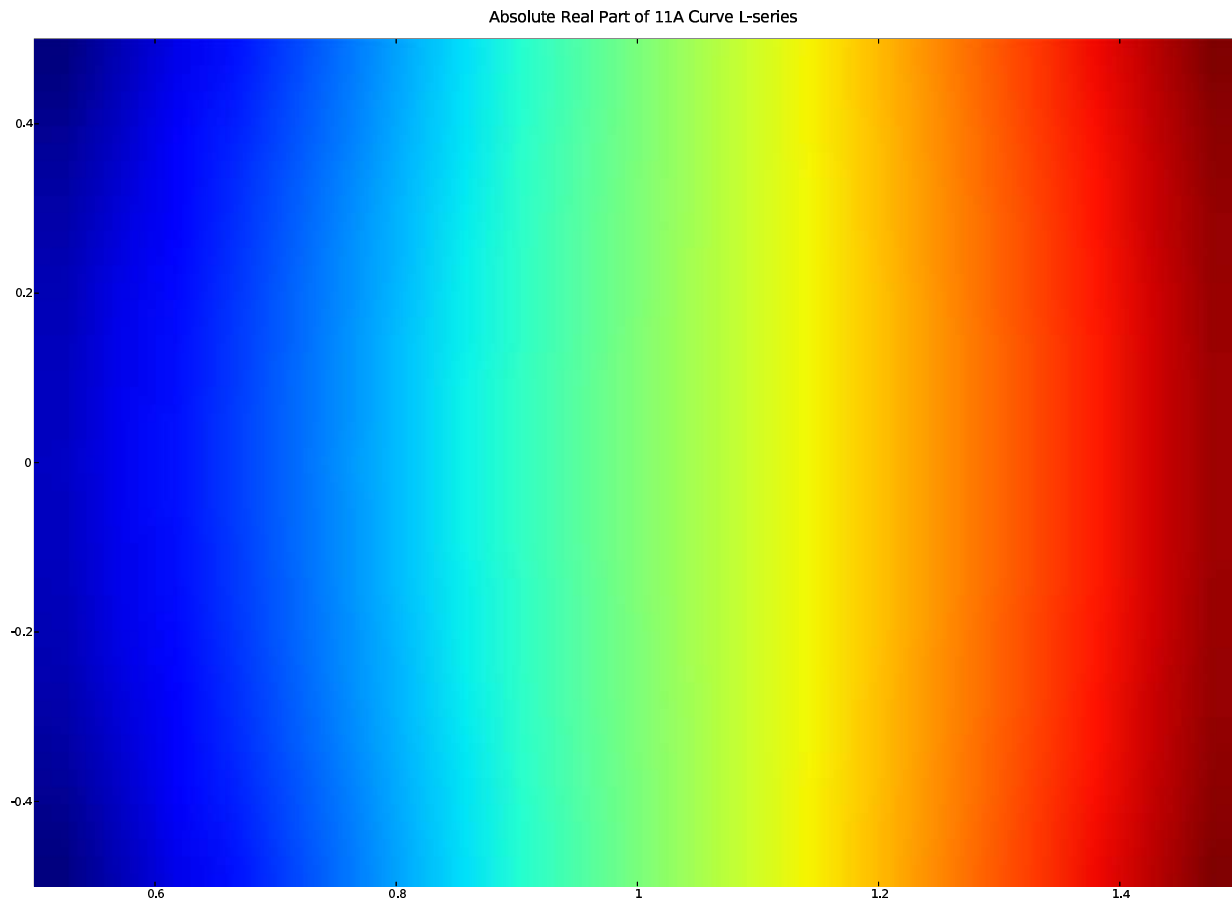Absolute Imaginary Part of 11A Curve Modular Form
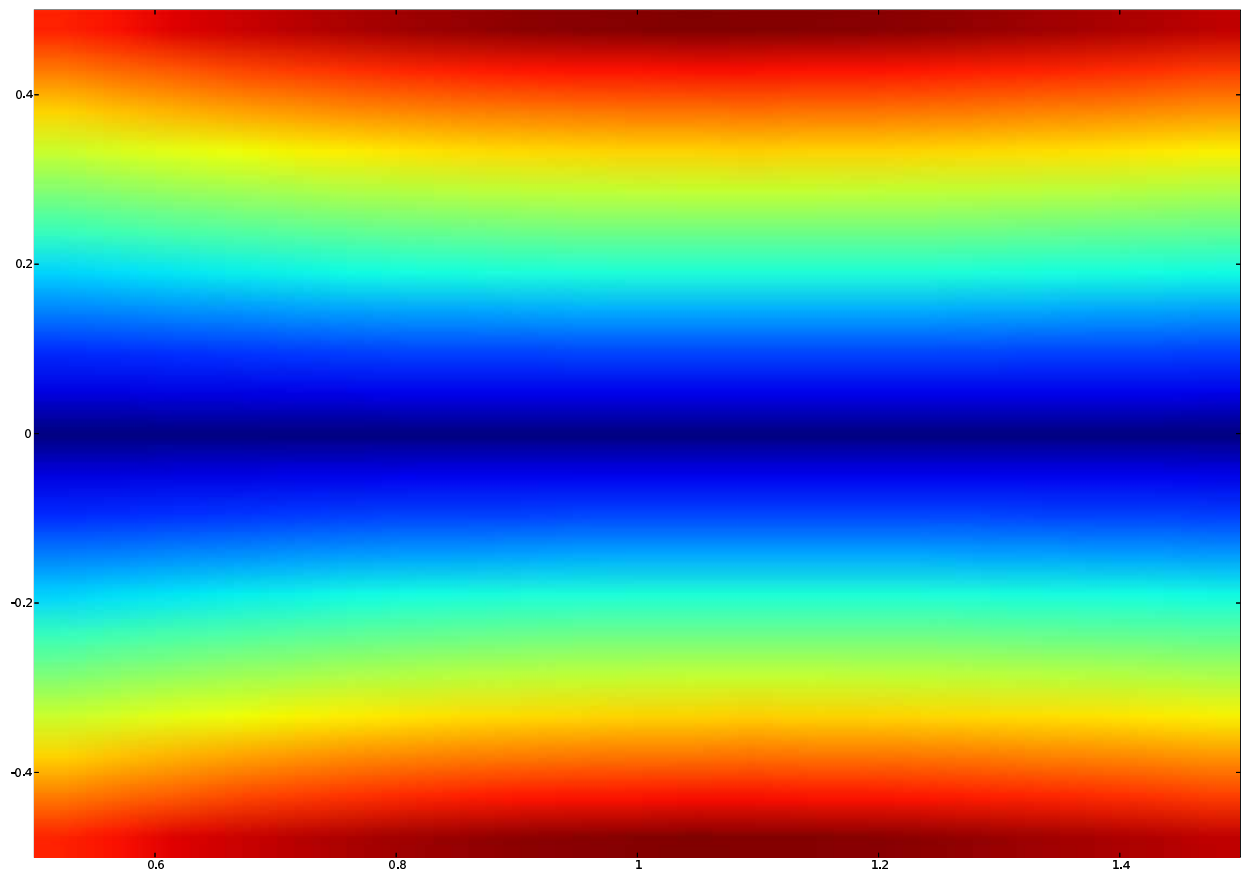
1.2 The *L*-series

The *L*-series is

$$\frac{1}{n} - \frac{2}{n^2} - \frac{1}{n^3} + \frac{2}{n^4} + \frac{1}{n^5} + \frac{2}{n^6} - \frac{2}{n^7} - \frac{2}{n^9} + \cdots .$$

Absolute Value of Elliptic Curve 11A Lseries Function

Absolute Real Part of 11A Curve L-series

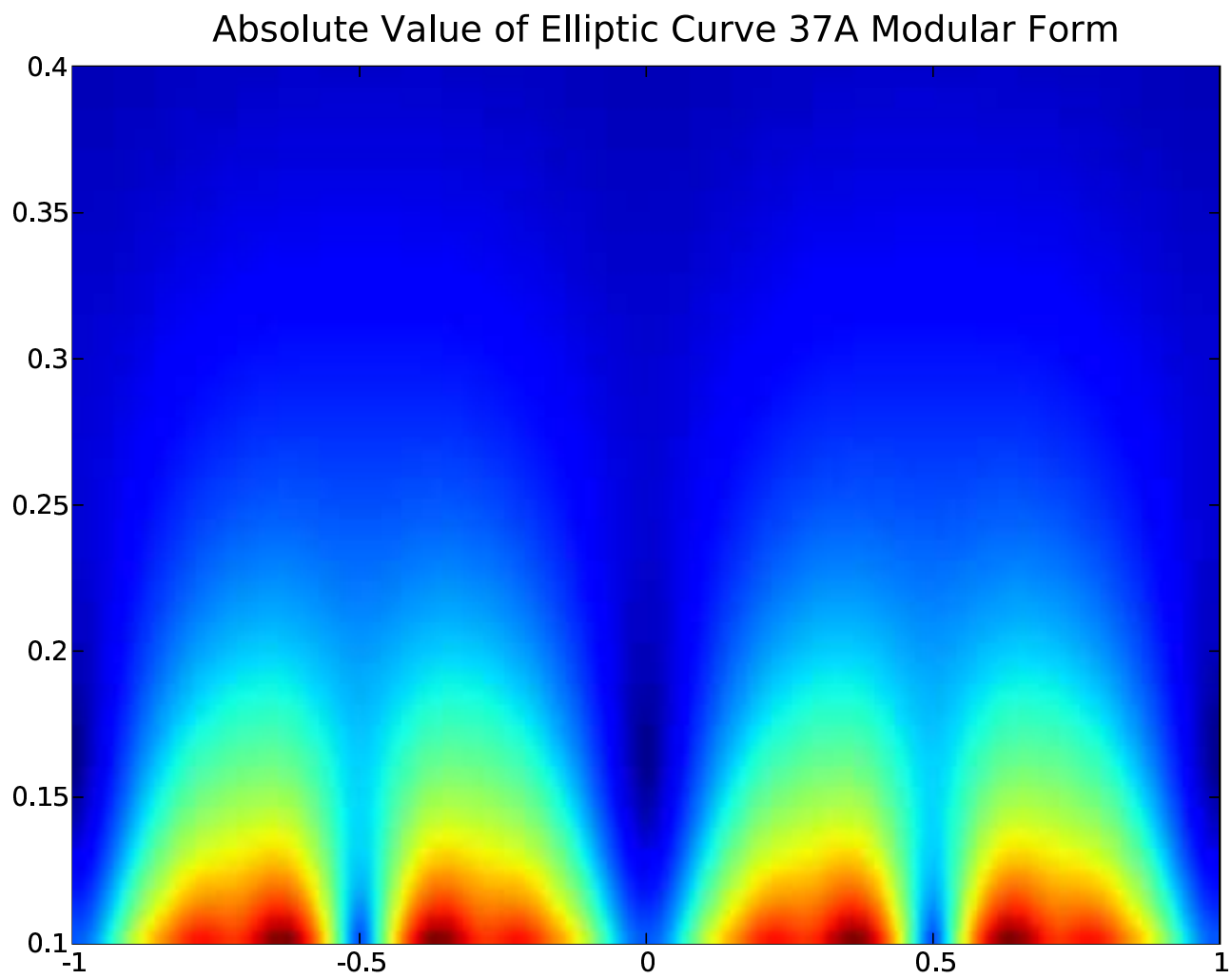Absolute Imaginary Part of 11A Curve L-series
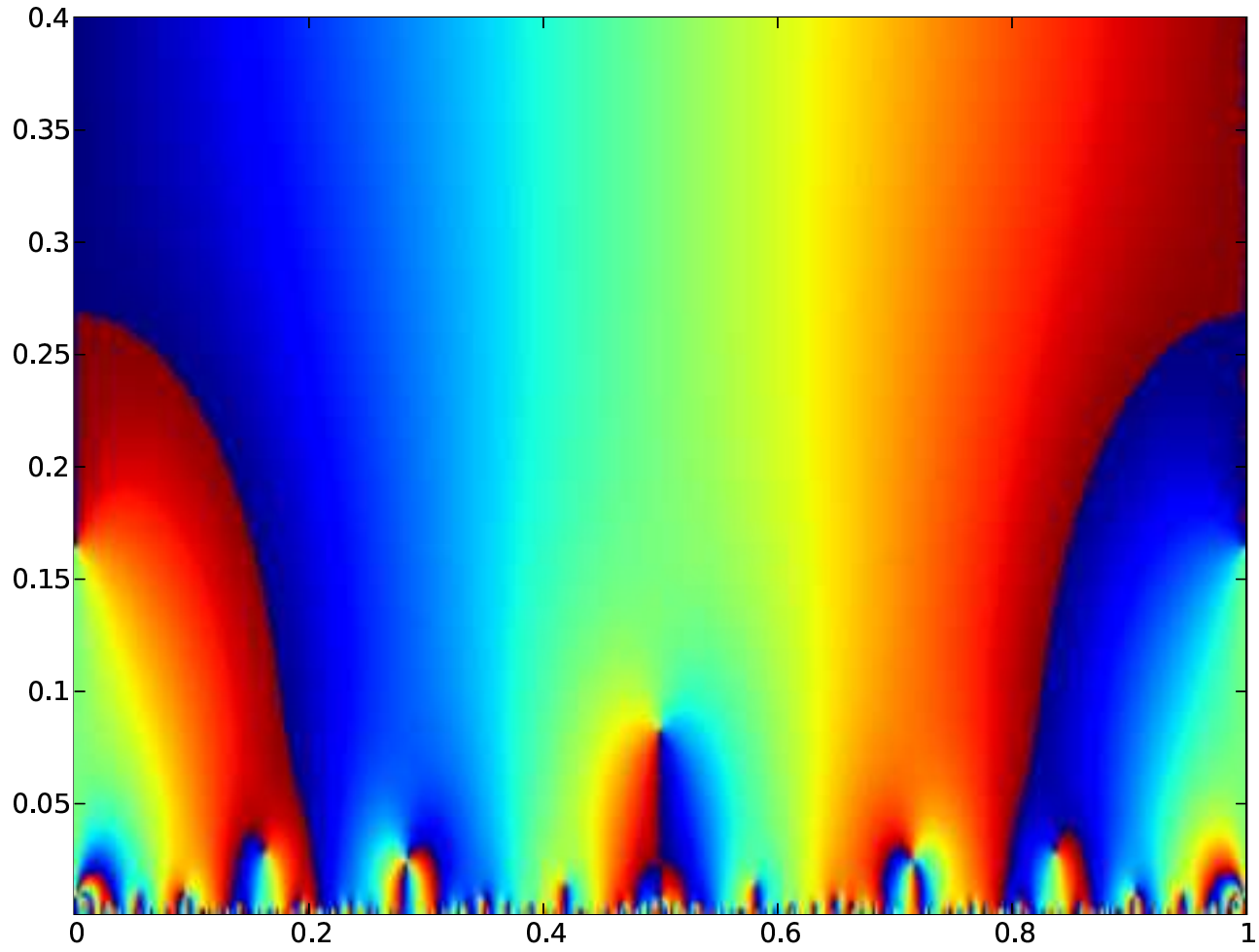
2 The Elliptic Curve $37A$

$$y^2 + y = x^3 - x$$

This is an elliptic curve of rank 1. The modular form is

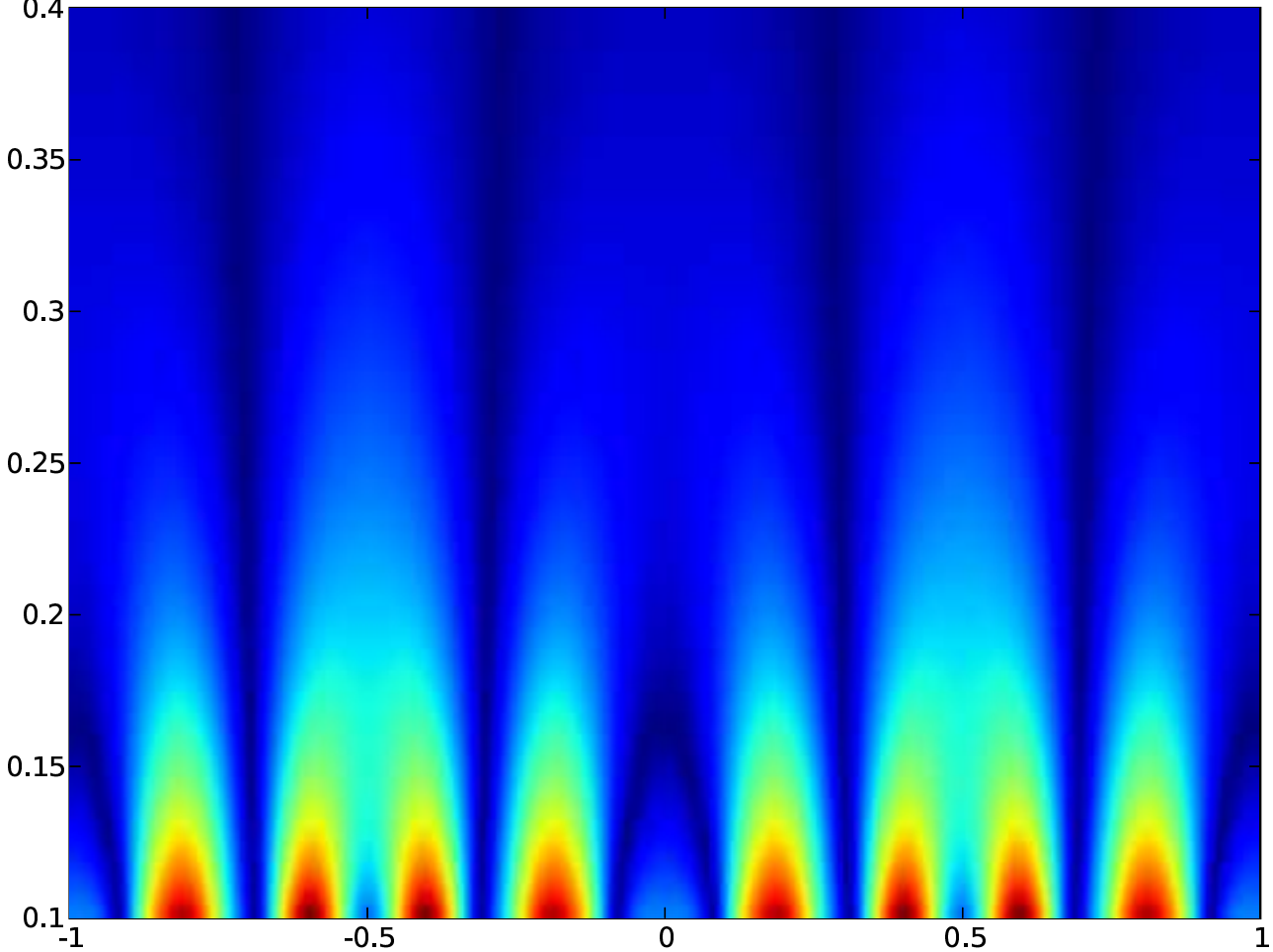$$q - 2q^2 - 3q^3 + 2q^4 - 2q^5 + 6q^6 - q^7 + 6q^9 + \cdots$$

2.1   The Modular Form

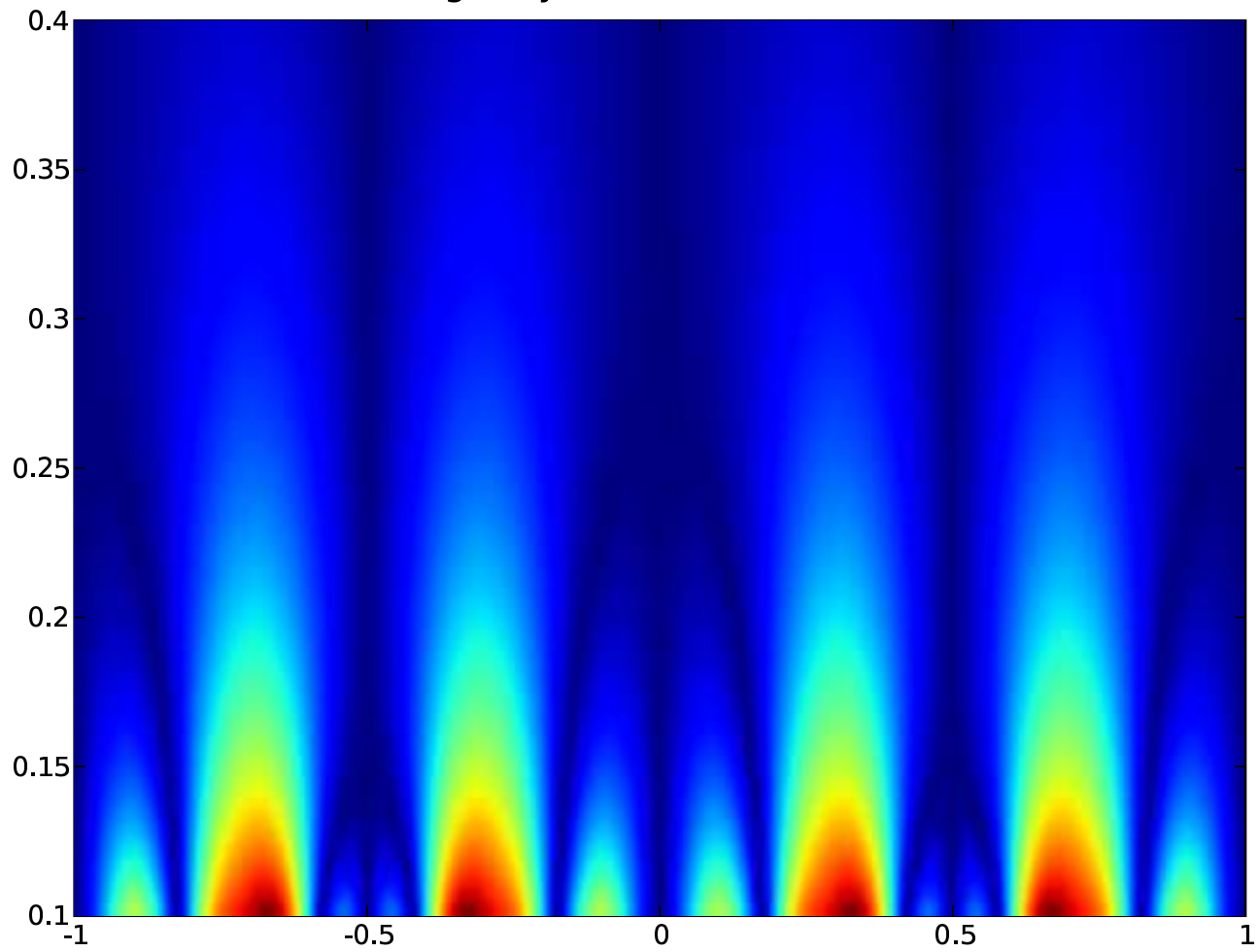Absolute Value of Elliptic Curve 37A Modular Form

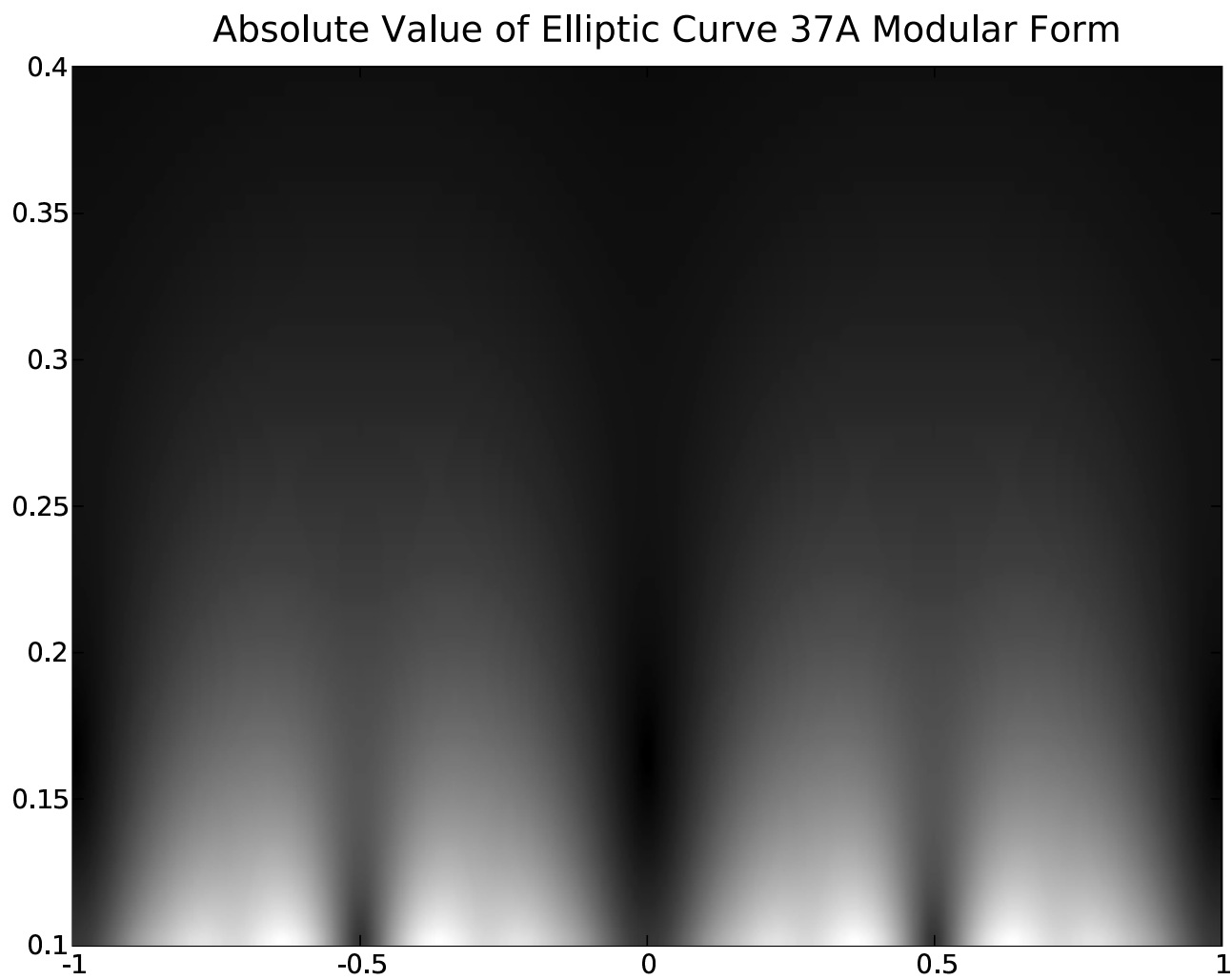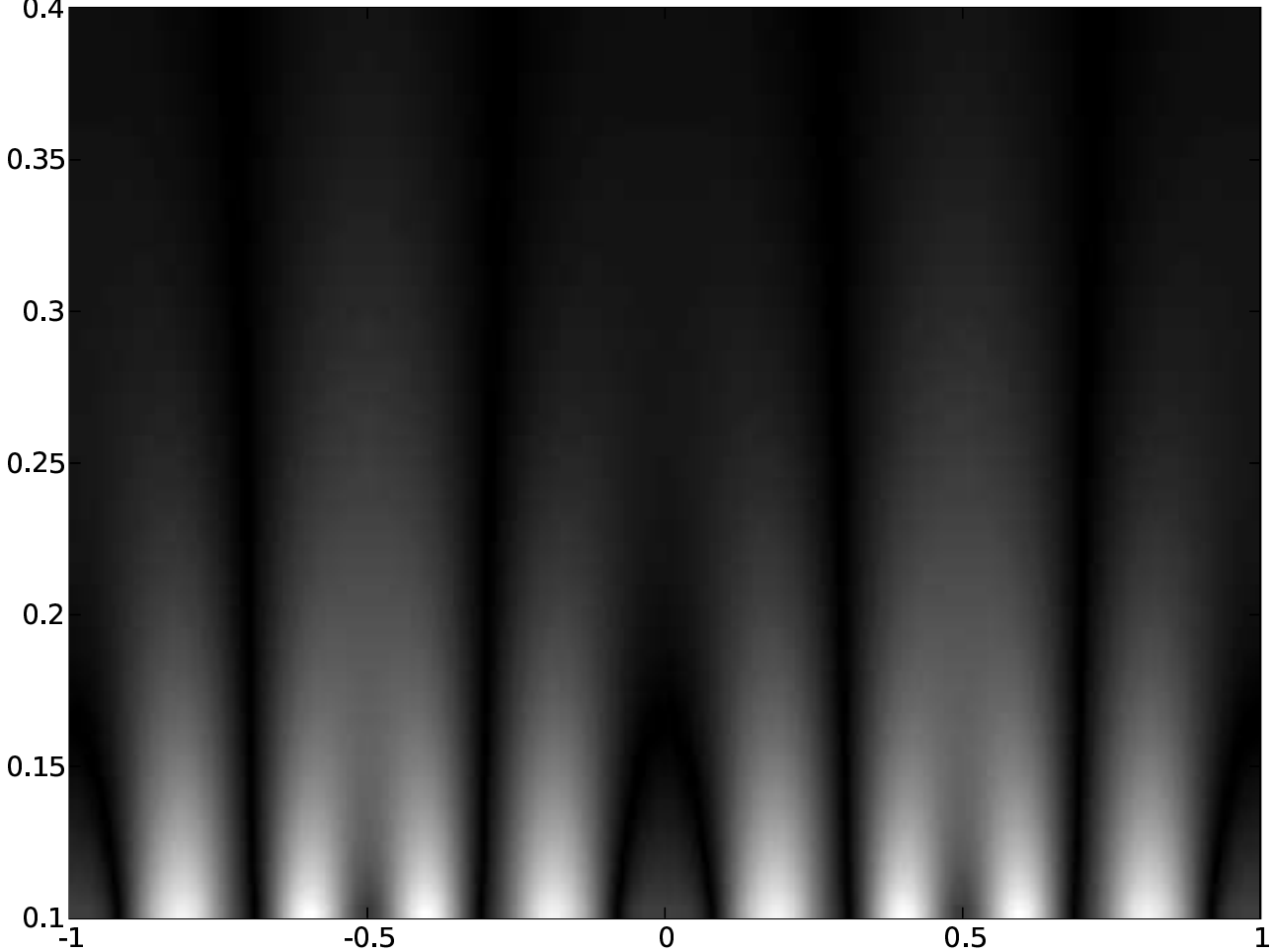Argument of Elliptic Curve 37A Modular Form

Absolute Real Part of 37A Curve Modular Form

Absolute Imaginary Part of 37A Curve Modular Form

2.2 Greyscale Versions of the Same

Absolute Value of Elliptic Curve 37A Modular Form

Absolute Real Part of 37A Curve Modular Form

Absolute Imaginary Part of 37A Curve Modular Form

2.3 The L-series
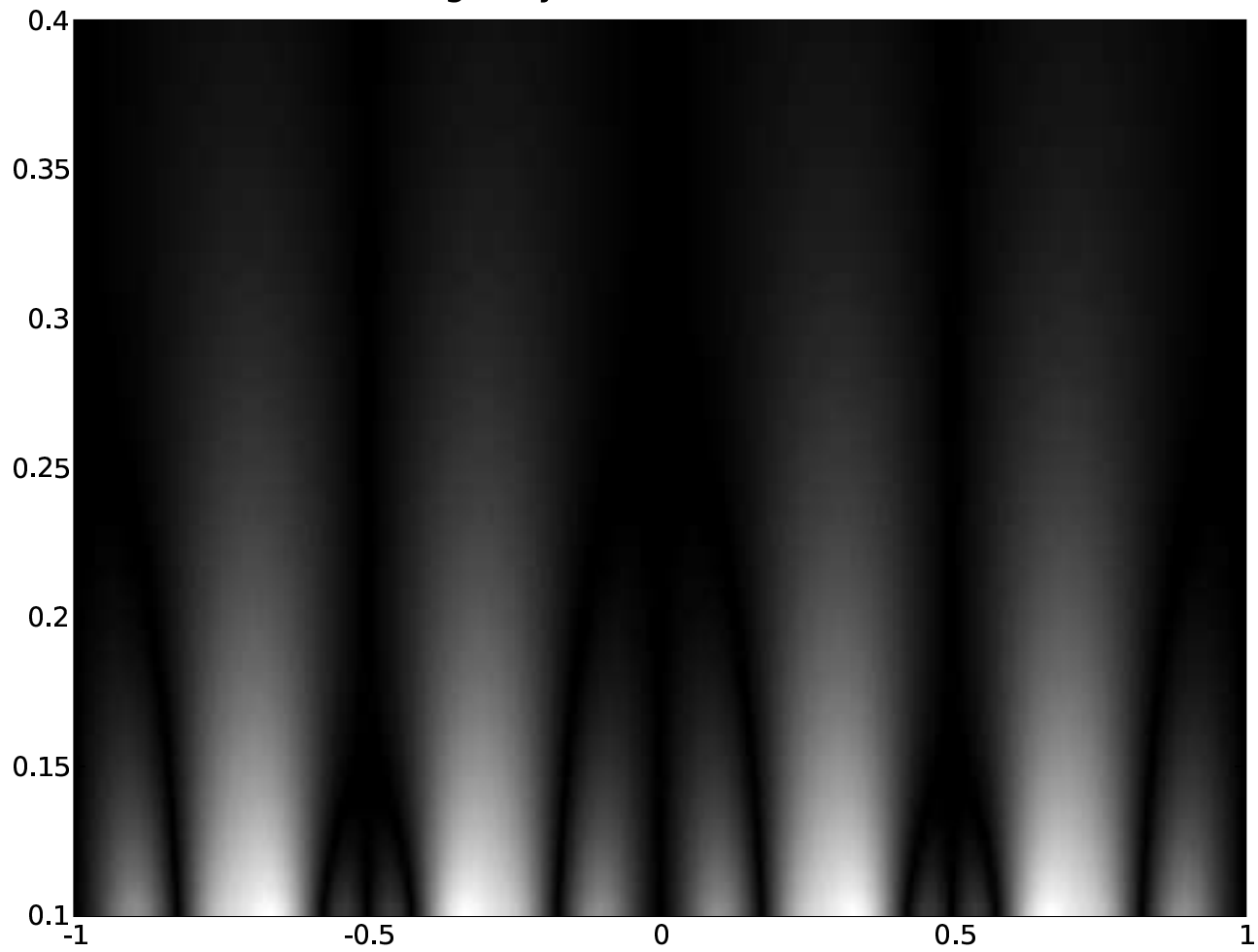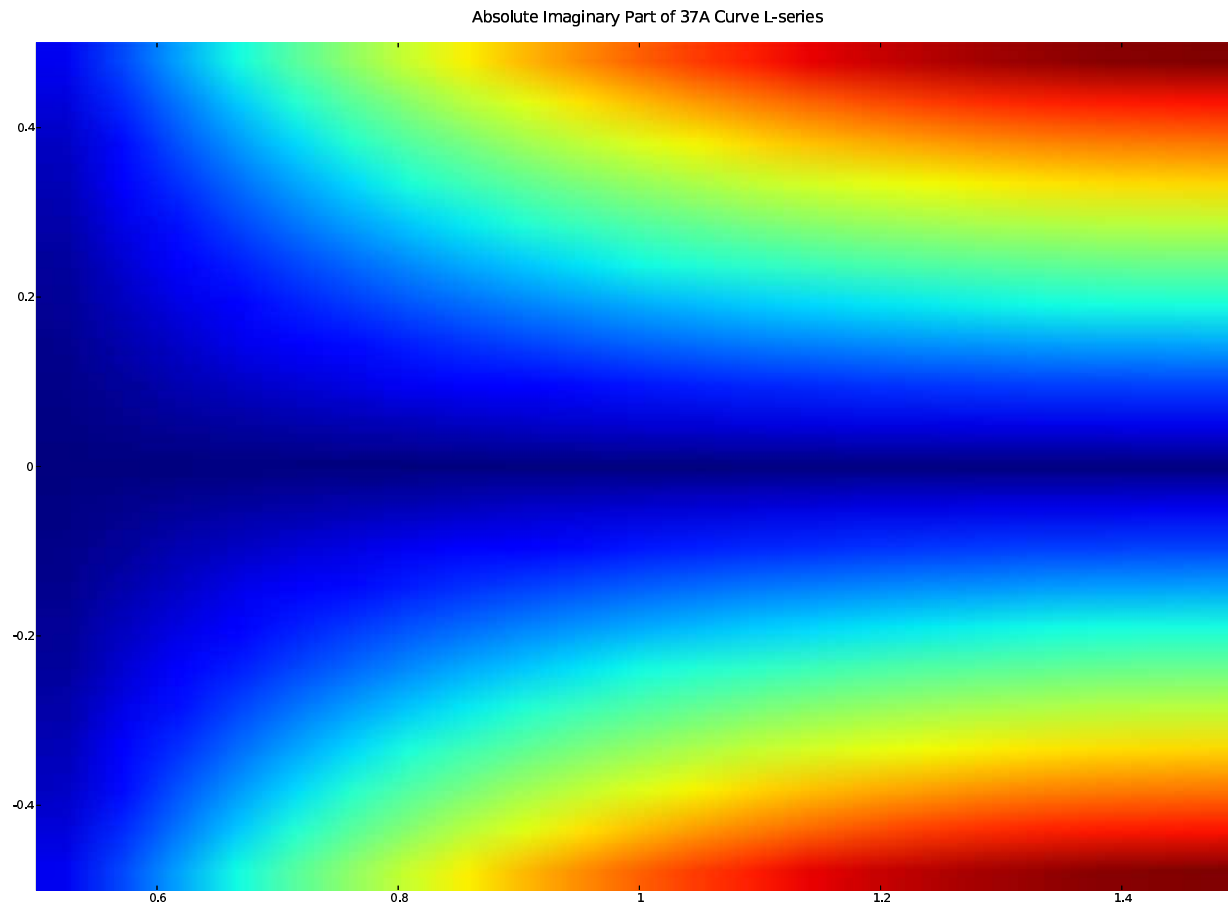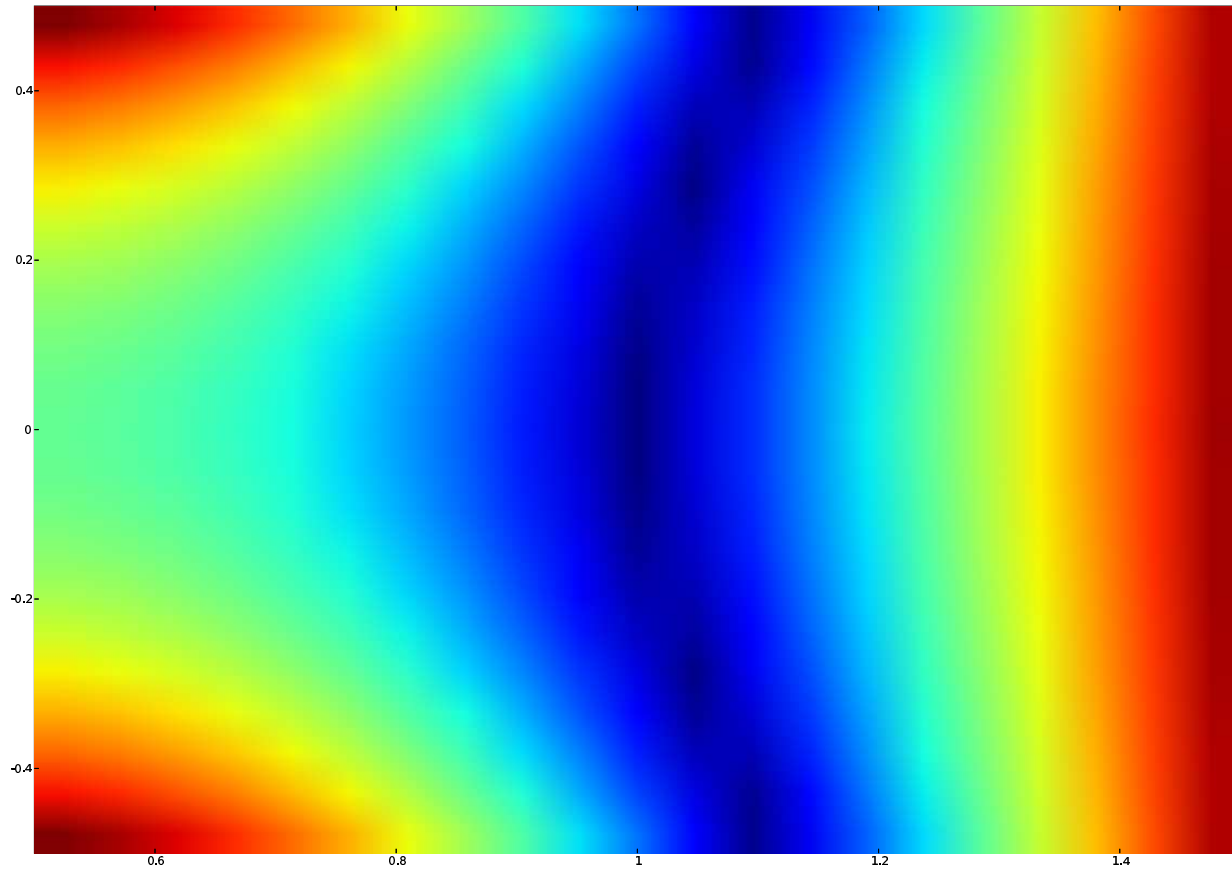
Absolute Imaginary Part of 37A Curve L-series

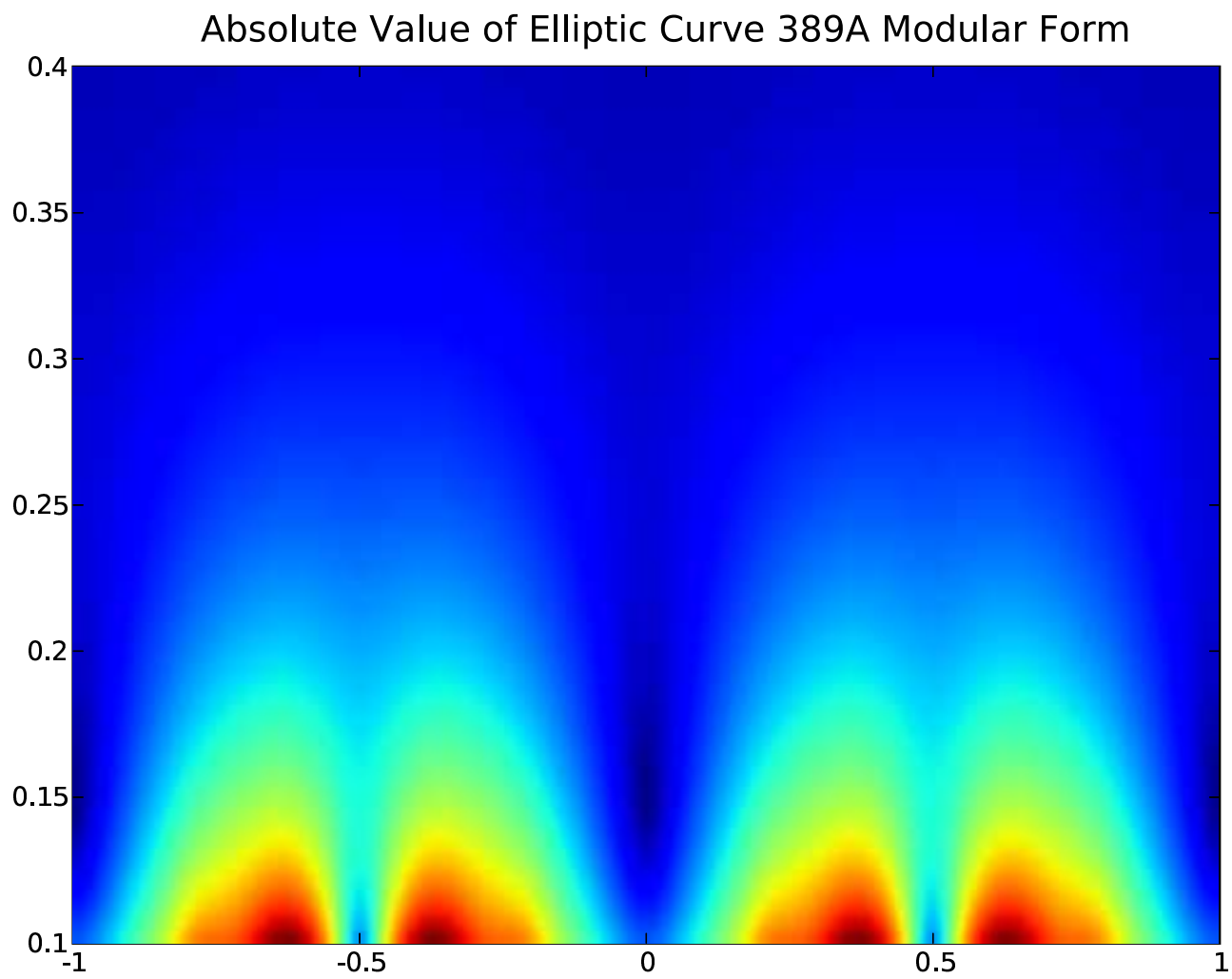Absolute Real Part of 37A Curve L-series

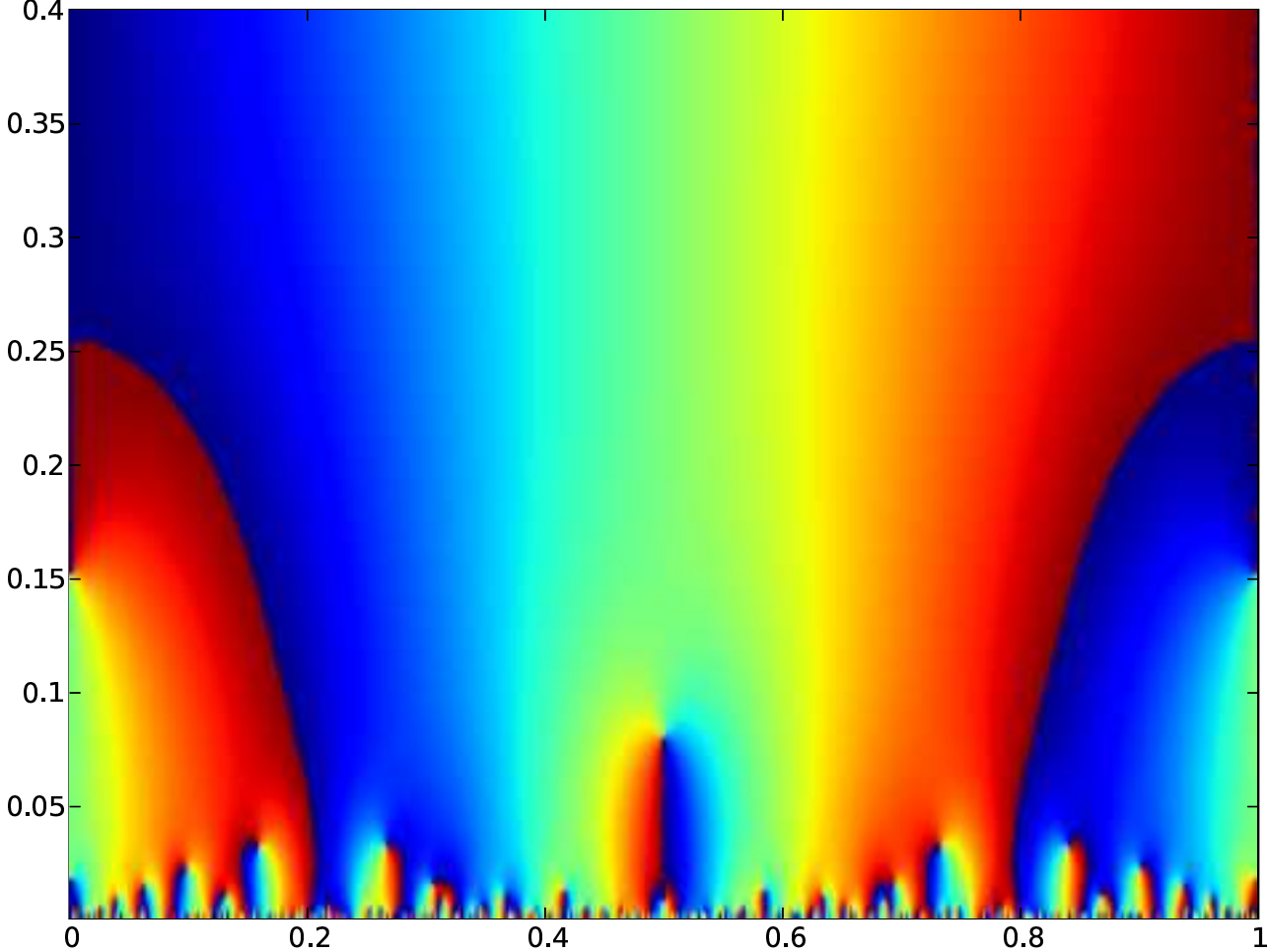# 3   The Elliptic Curve $389A$

$$y^2 + y = x^3 + x^2 - 2x$$

This is an elliptic curve of rank 2. The modular form is

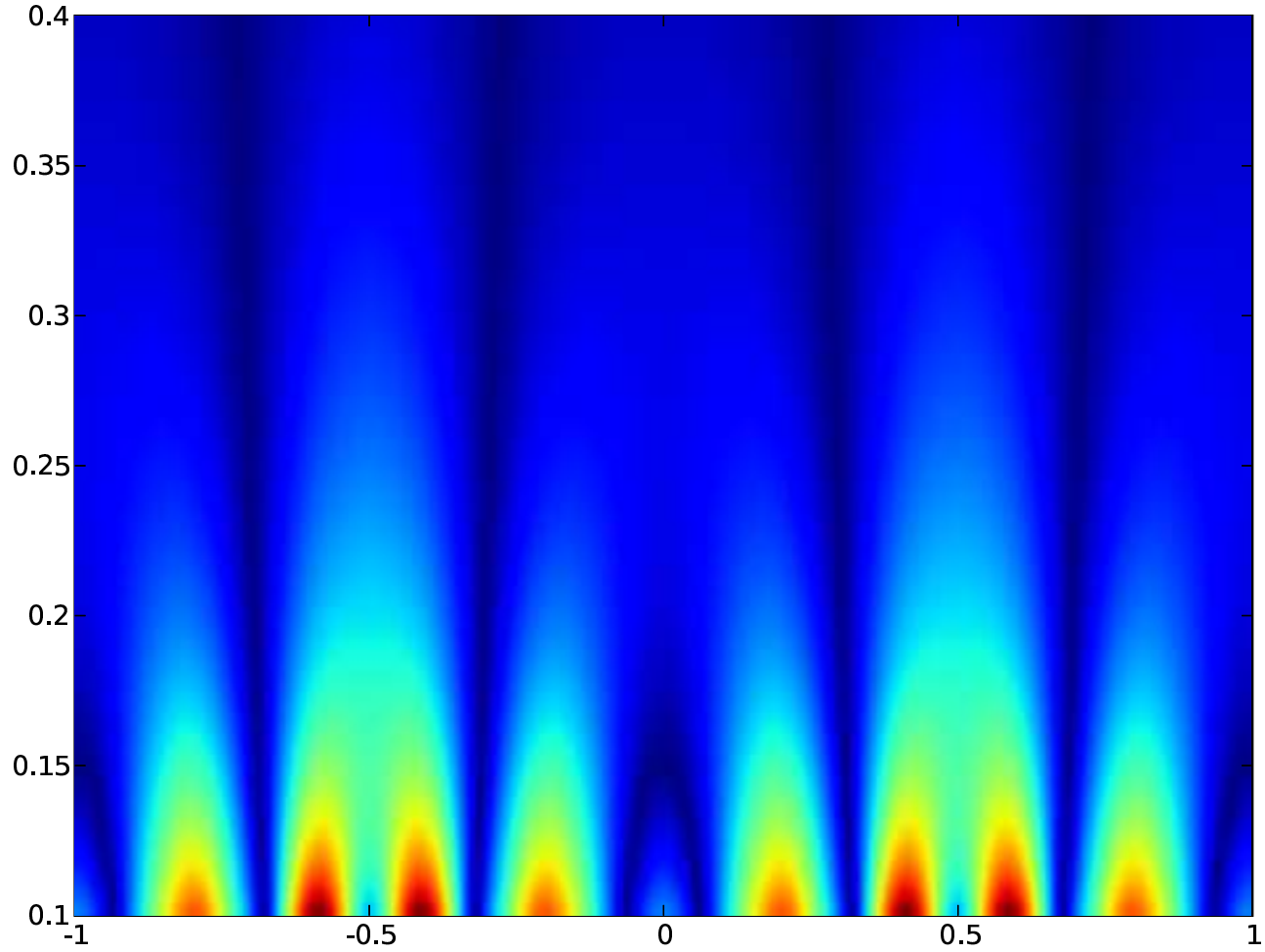$$q - 2q^2 - 2q^3 + 2q^4 - 3q^5 + 4q^6 - 5q^7 + q^9 + \cdots$$
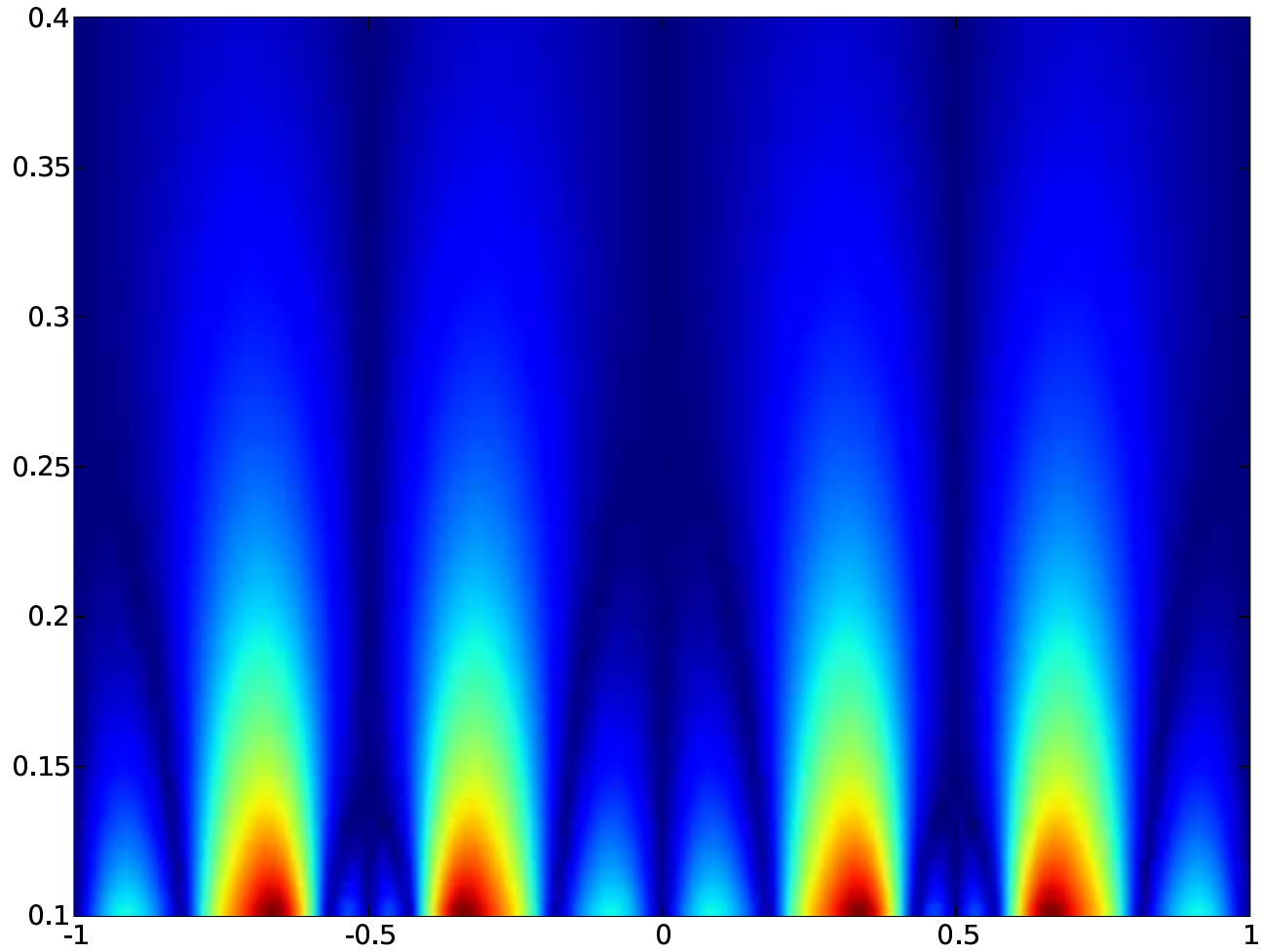
3.1   The Modular Form
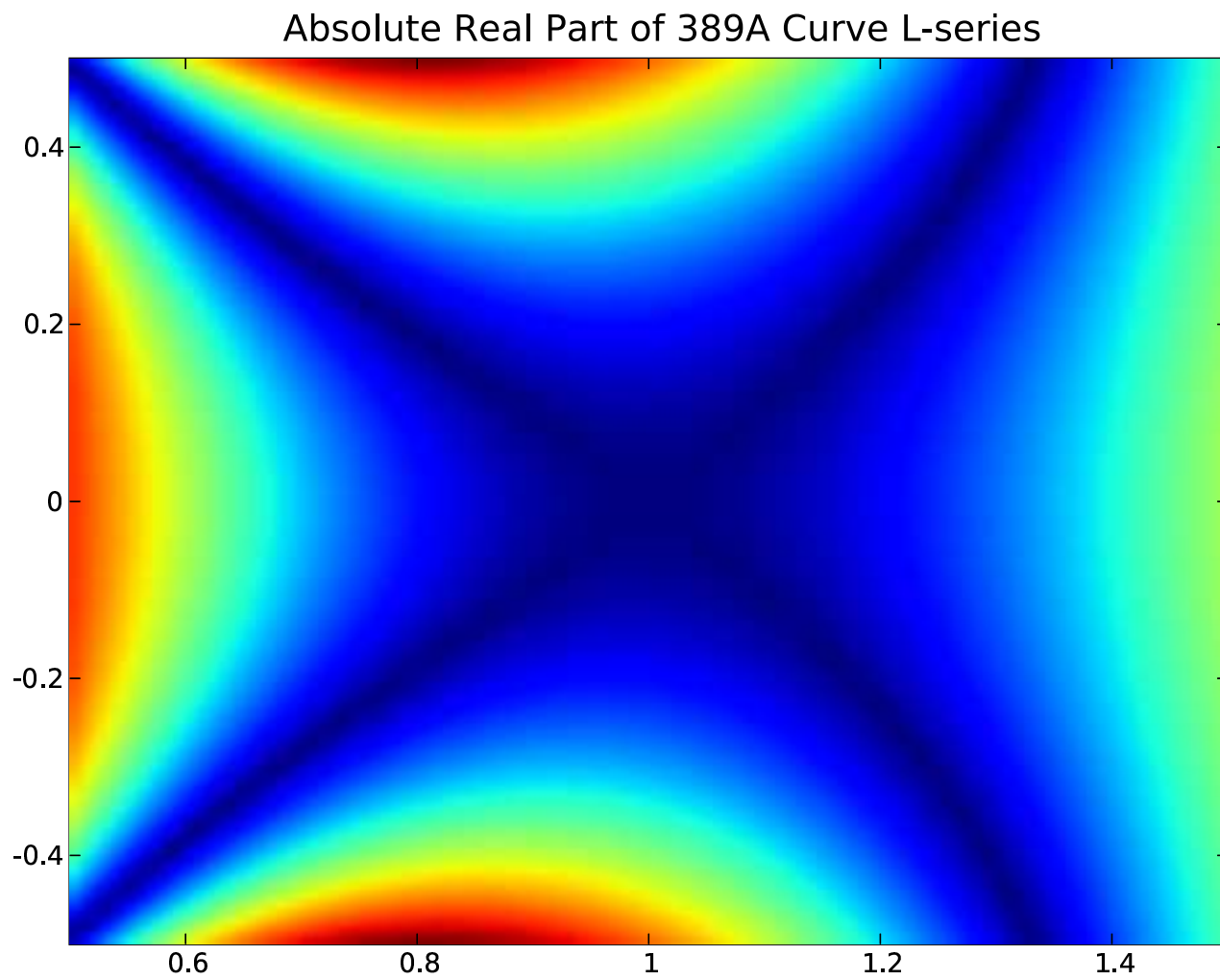

Absolute Value of Elliptic Curve 389A Modular Form

Argument of Elliptic Curve 389A Modular Form

Absolute Real Part of 389A Curve Modular Form

Absolute Imaginary Part of 389A Curve Modular Form

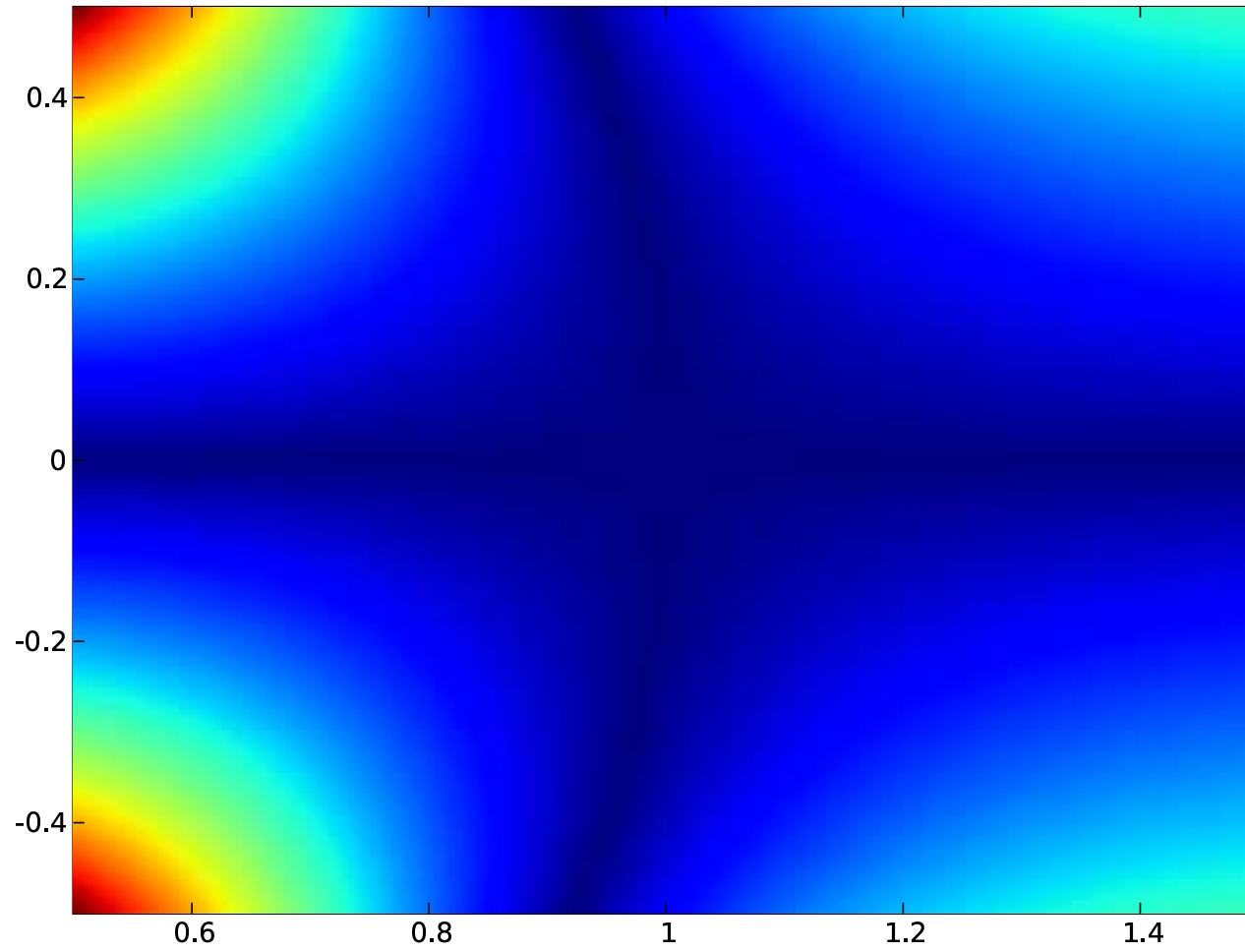**Absolute Real Part of 389A Curve L-series**

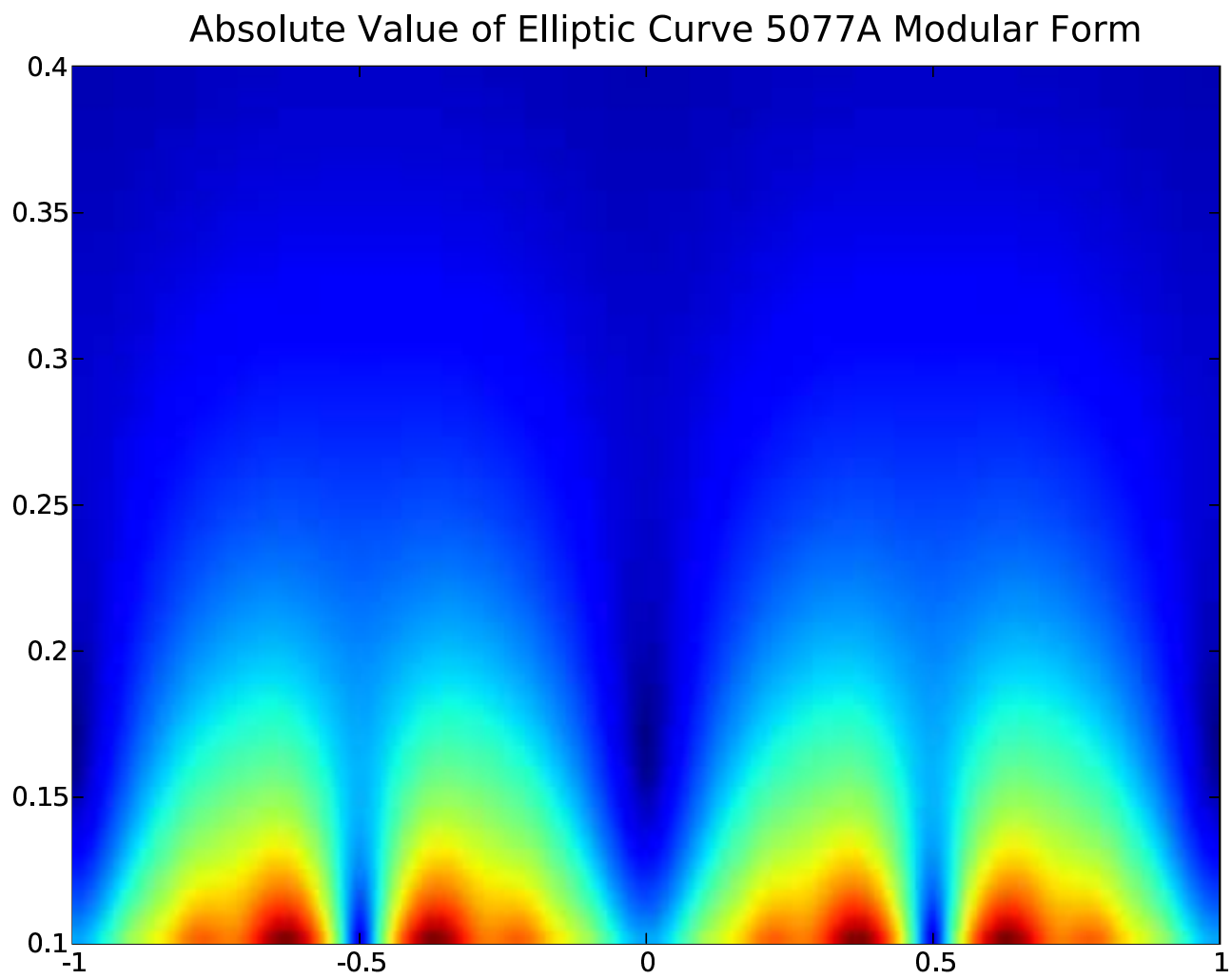Absolute Imaginary Part of 389A Curve L-series

4   The Elliptic Curve $5077A$
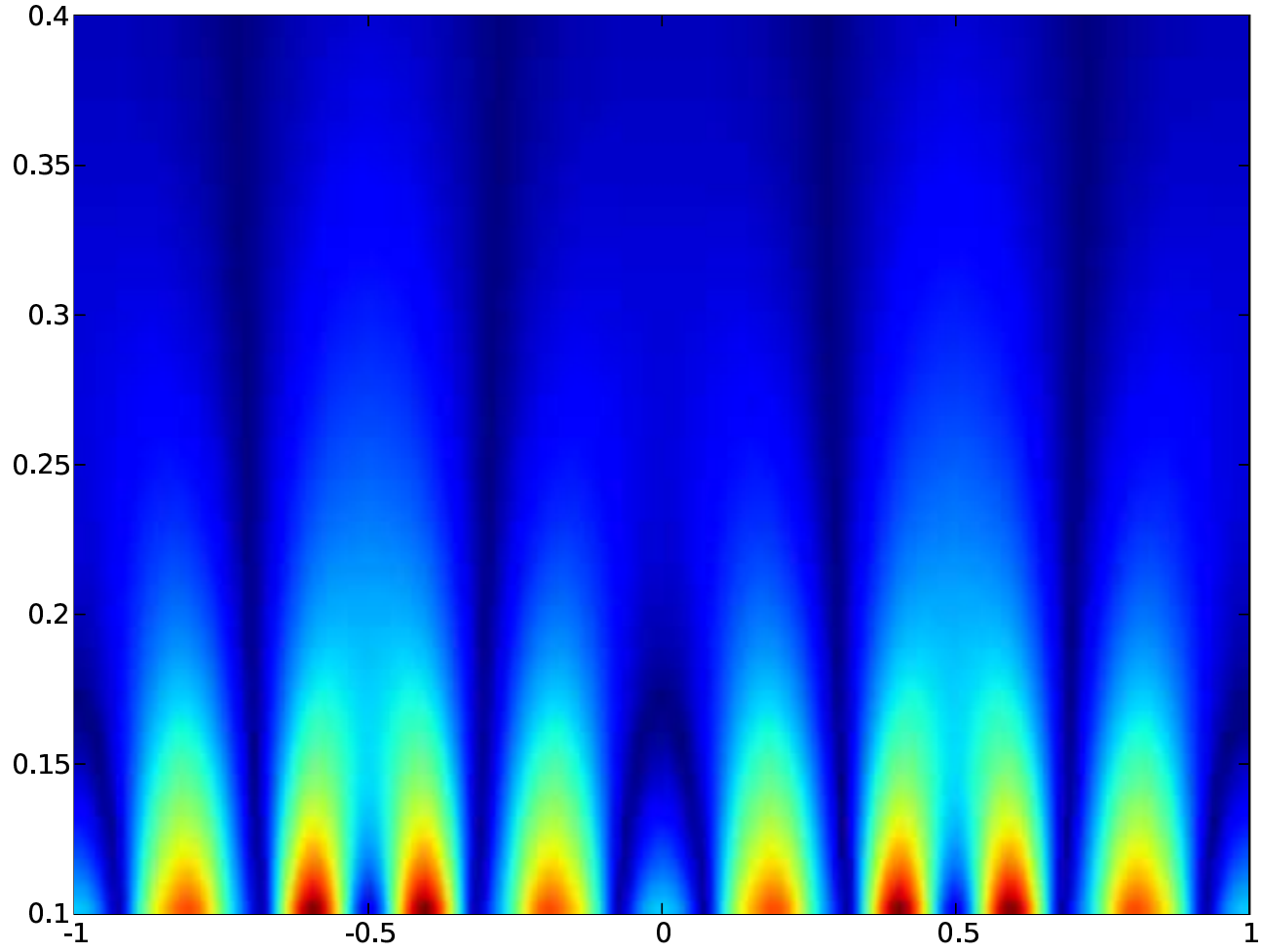
$$y^2 + y = x^3 - 7x + 6$$

This is an elliptic curve of rank 3. The modular form is

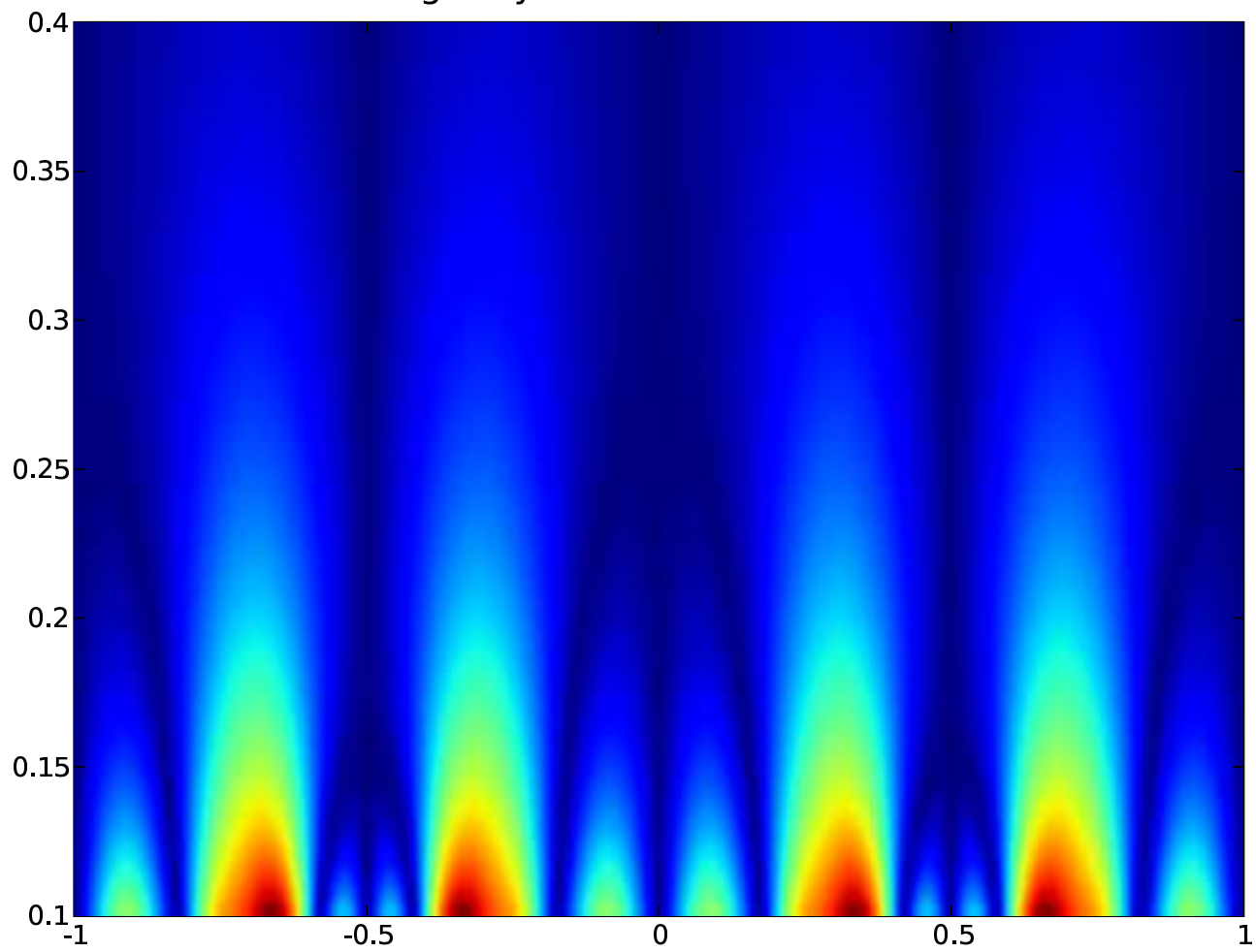$$q - 2q^2 - 3q^3 + 2q^4 - 4q^5 + 6q^6 - 4q^7 + 6q^9 + \cdots$$

4.1 The Modular Form

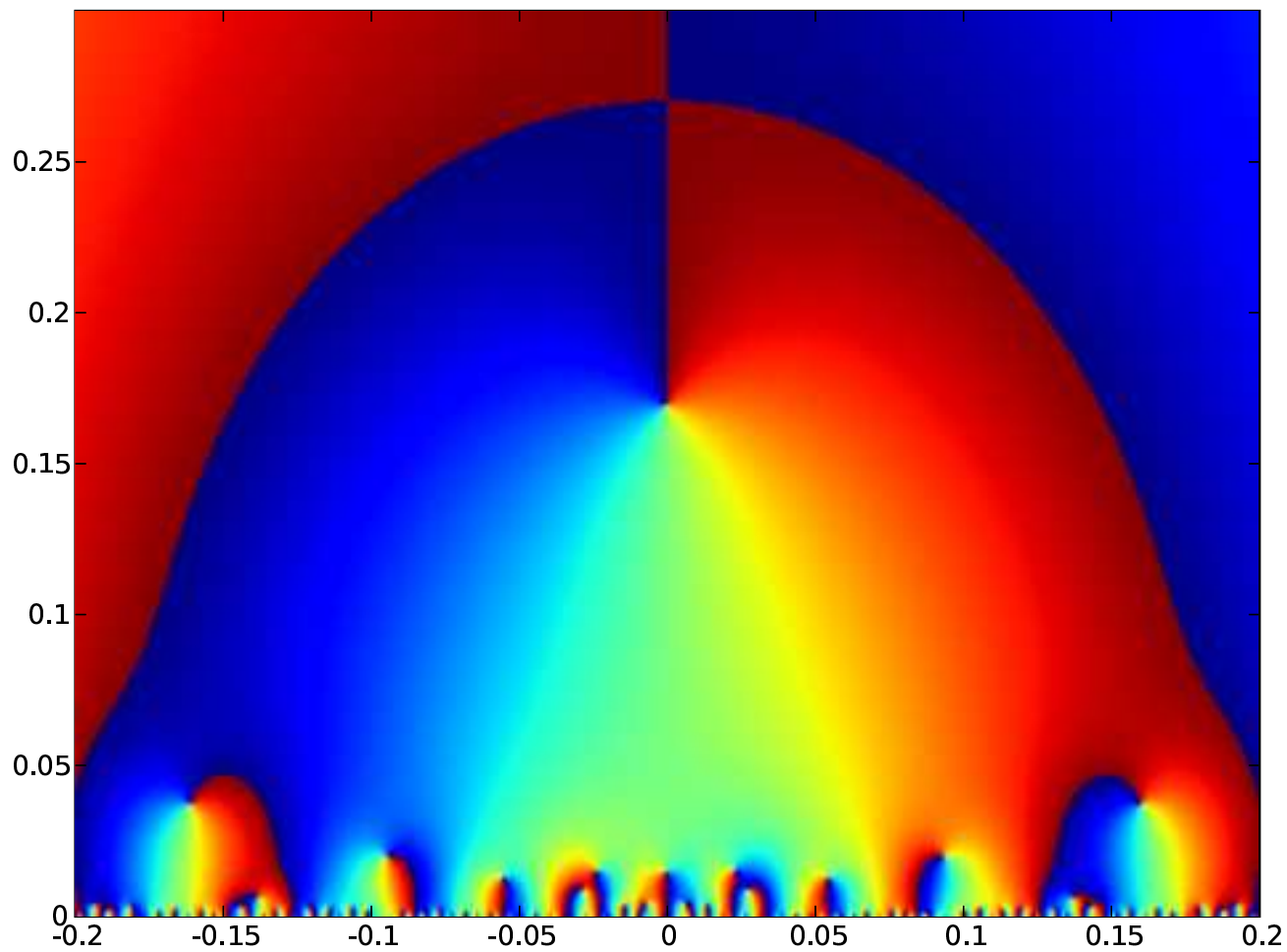Absolute Value of Elliptic Curve 5077A Modular Form

Absolute Real Part of 5077A Curve Modular Form

Absolute Imaginary Part of 5077A Curve Modular Form

Argument of Elliptic Curve 5077A Modular Form

4.2 The L-series

Absolute Real Part of 5077A Curve L-series