Harvard Math 129: Algebraic Number Theory
Homework Assignment 8

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Due: THURSDAY, April 21, 2005

The problems have equal point value, and multi-part problems are of the same value. In any problem where you use a computer, include in your solution the exact commands you type and their output. You may use any software, including (but not limited to) MAGMA and PARI.

1. Look up the Riemann-Roch theorem in a book on algebraic curves.
   (a) Write it down, somewhat in your own words.
   (b) Let E be an elliptic curve over a field K. Use the Riemann-Roch theorem to deduce that the natural map
   $$E(K) \to \mathrm{Pic}^0(E/K)$$
   is an isomorphism. (If you can't figure out the argument because you aren't very familiar with algebraic curves, try to find it in a book and put it in your own words—this is not cheating.)
2. Write down a "proposal" or summary of your final project. Do something reasonable here for full credit.